Probabilistic Performance-based Earthquake Engineering

KHALID M. MOSALAM, PROFESSOR

STRUCTURAL ENGINEERING, MECHANICS & MATERIALS

DEPARTMENT OF CIVIL & ENVIRONMENTAL ENGINEERING

UNIVERSITY OF CALIFORNIA, BERKELEY

ACKNOWLEDGEMENTS:

- FIB TASK GROUP 7.7: PROBABILISTIC PERFORMANCE-BASED SEISMIC DESIGN
- DR. SELIM GÜNAY, UC-BERKELEY

Course Outline 1/2

Part I:

1. PBEE assessment methods
   - ✓ Conditional probability approaches such as SAC/FEMA & PEER formulations
   - ✓ Unconditional probabilistic approach

Questions

2. PBEE design methods
   - ✓ Optimization-based methods
   - ✓ Non optimization-based methods

Questions

3. PEER PBEE formulation demonstrated for electric substation equipment
   - ✓ Introduction
   - ✓ Hazard analysis
   - ✓ Structural analysis
   - ✓ Damage analysis
   - ✓ Loss analysis
   - ✓ Combination of analyses

Questions

Course Outline 2/2

Part II:

1. Application 1: Evaluation of the effect of unreinforced masonry infill walls on reinforced concrete frames with probabilistic PBEE

Questions

2. Application 2: PEER PBEE assessment of a shearwall building located on the University of California, Berkeley campus

Questions

3. Application 3: Evaluation of the seismic response of structural insulated panels with probabilistic PBEE

Questions

4. Future extension to multi-objective performance-based sustainable design
5. Recapitulation

I-1 PBEE Assessment Methods

KHALID M. MOSALAM, PROFESSOR

UNIVERSITY OF CALIFORNIA, BERKELEY

Outline

1. **Conditional Probabilistic Approach**
   - **Introduction**
   - **SAC/FEMA**
   - **PEER PBEE (very brief)**

2. **Unconditional Probabilistic Approach**

Conditional Probabilistic Approach: Introduction

- Aimed to be practice-oriented
  - Currently employed mostly in the academic community
  - Expected to gain increasing acceptance in practice in near future

- Common standpoint of the methods: Use of intensity measure (IM) as an interface between seismology and structural engineering
  - IM is commonly represented with a hazard curve
  - *Structural engineers* need to have basic information on how to obtain a hazard curve, otherwise end up with incorrect hazard representation

Excellent Review Article: Why Do Modern Probabilistic Seismic-Hazard Analyses Often Lead to Increased Hazard Estimates? By J.J. Bommer and N.A. Abrahamson [*Bulletin of the Seismological Society of America*, **96**(6):1967–1977, Dec. 2006]

Conditional Probabilistic Approach: SAC[1]/FEMA[2]

- During 1994 Northridge earthquake, some steel-moment-resisting-frame (SMRF) buildings underperformed by fractures in many beam-column joints which were supposed to remain elastic

Column Fracture in Beam Column Testing (courtesy of M. Engelhart)

- Originally developed for investigation of this unexpected behavior and assessment of the seismic performance of these SMRF buildings

- Applicable to all building types with adjustments

(1)SAC is a joint venture of the **S**tructural Engineers Association of California (SEAOC), the **A**pplied Technology Council (ATC), and **C**alifornia Universities for Research in Earthquake Engineering (CUREE), formed to address both immediate and long-term needs related to solving the problem of the WSMF connection.

(2)US **F**ederal **E**mergency **M**anagement **A**gency (FEMA) www.fema.gov

Conditional Probabilistic Approach: SAC/FEMA

- An empirical method based on assuming that an engineer uses ground motions and a computational model of a structure to test the likelihood that a building will perform as intended (**?**) over the period of interest

System Performance

Hazard Levels (Return Period)	Fully Operational	Operational	Life Safety	Near Collapse
Frequent (43 years)	●	o	o	o
Occasional (72 years)	Δ	●	o	o
Rare (475 years)	♦	Δ	●	o
Very rare (949 years)		♦	Δ	●

o: unacceptable performance

●: basic safety objective

Δ: essential hazardous objective

♦: safety critical objective

Ex. 1: Is the structure capable of remaining fully operational in a frequent earthquake ?

Ex. 2: Is the structure capable of surviving a very rare earthquake?

Conditional Probabilistic Approach: SAC/FEMA

- Can be considered as a special application of the more general PEER PBEE framework (***to be discussed later!***)
  - Complete consideration of uncertainty & probability
  - Performance assessment not with decision variables (DV)
  - Performance assessment considering
    - Intensity Measure (IM)
    - Engineering Demand Parameter (EDP)
    - Capacity of the Engineering Demand Parameter (ECP)
  - DV can be interpreted as a *binary* indicator of achieving the performance level:
    - **0**: unacceptable performance
    - **1**: acceptable performance

Conditional Probabilistic Approach: SAC/FEMA

Motivation for Consideration of Uncertainty

Traditional earthquake design (TED) philosophy:

- Prevent damage in low-intensity EQ (**50% in 50 years**)
- Limit damage to repairable levels in medium-intensity EQ (**10% in 50 years**)
- Prevent collapse in high-intensity EQ (**2% in 50 years**)

- If an engineer would accept that the world is deterministic, then if one observes a structure not collapsing for the 2% in 50 years event, one *incorrectly* concludes that the probability of global collapse of the building is certainly < 2% in 50 years

- There are many sources of uncertainty in this problem that need to be taken into account for a realistic assessment of the collapse probability of this building

- These uncertainties will probably make the probability of global collapse much higher than 2% in 50 years

Conditional Probabilistic Approach: SAC/FEMA

Types & Sources of Uncertainty

Alea* (Latin)=*Dice

Aleatory uncertainty (*randomness*): The uncertainty inherent in a nondeterministic (stochastic, random) phenomenon.

Examples: 1) The location and the magnitude of the next earthquake; 2) The intensity of the ground shaking generated at a given site

Epist (Greek): Knowledge

Epistemic uncertainty: The uncertainty attributable to incomplete knowledge about a phenomenon that affects our ability to model it.

Example: The definition of parameters & rules of a constitutive model for concrete

Conditional Probabilistic Approach: SAC/FEMA

Background

Total probability theorem:

Given n mutually exclusive events* $A_1, \ldots, A_n$ whose probabilities sum to 1.0, then the probability of an arbitrary event B:

$$p(B) = p(B|A_1)\,p(A_1) + p(B|A_2)\,p(A_2) + \ldots + p(B|A_n)\,p(A_n)$$

$$p(B) = \sum_i \left[\underbrace{p(B|A_i)}_{\text{Conditional probability of B given the presence of } A_i} \; \underbrace{p(A_i)}_{\text{Probability of } A_i} \right]$$

*Occurrence of any one of them automatically implies the non-occurrence of the remaining n−1 events

Conditional Probabilistic Approach: SAC/FEMA

Background

- Calculate the probability of exceedance (POE), **P**, of the ith value of EDP

$$P\left(EDP^i\right) = \sum_m \underbrace{P\left(EDP^i \middle| IM_m\right)}_{\text{Computational model \& simulations}} \underbrace{p\left(IM_m\right)}_{\text{Hazard curve}}$$

- Calculate the probability (**p**) of EDP^i

$$\text{for } i = 1:\#\text{ of EDP values}$$

$$p(EDP^i) = P(EDP^i) \quad \text{if } i = \#\text{ of EDP values}$$

$$p(EDP^i) = P(EDP^i) - P(EDP^{i+1}) \quad \text{otherwise}$$

Conditional Probabilistic Approach: SAC/FEMA

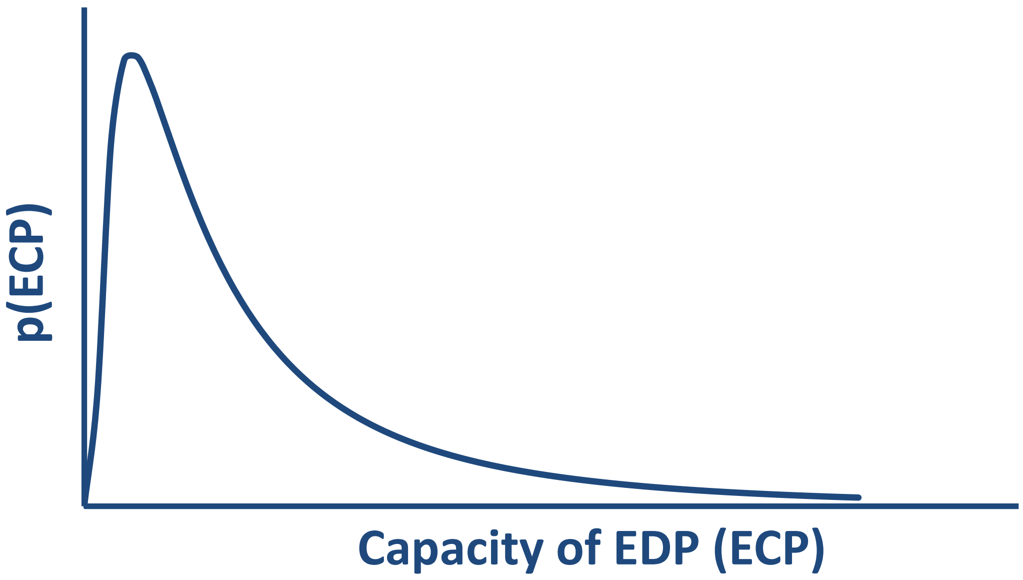
Background

- If an engineer is sure that the structure would fail its performance level when it reaches a certain *limiting* EDP value (EDP^L) → the probability of not meeting that performance level (p_{fPL}) = $P(EDP^L)$
- However, the engineer cannot be sure about the above issue, since there is uncertainty in the corresponding capacity limit
- Theoretically, every value of EDP has a finite likelihood of making a structure fails a performance level
- Uncertainty in the capacity of an EDP (ECP) should be considered for the calculating p_{fPL}
- Considering uncertainty in capacity: p_{fPL} is defined as the probability of ECP being smaller than EDP [p(ECP<EDP)]; Same uncertainty is considered in a different format in Damage Analysis stage of PEER PBEE framework

Conditional Probabilistic Approach: SAC/FEMA

Background

Uncertainty in capacity: Capacity of EDP that corresponds to a Performance Level (PL) is represented with a probability distribution



Conditional Probabilistic Approach: SAC/FEMA

Background

Table 6-7 Modeling Parameters and Numerical Acceptance Criteria for Nonlinear Procedures— Reinforced Concrete Beams

PR

Conditions			Modeling Parameters[3]: Plastic Rotation Angle, radians		Modeling Parameters[3]: Residual Strength Ratio	Acceptance Criteria[3]: Plastic Rotation Angle, radians — Performance Level	Component Type: Primary		Component Type: Secondary	
			a	b	c	IO	LS	CP	LS	CP
i. Beams controlled by flexure[1]										
$\frac{\rho-\rho'}{\rho_{bal}}$	Trans. Reinf.[2]	$\frac{V}{b_w d\sqrt{f'_c}}$								
≤ 0.0	C	≤ 3	0.025	0.05	0.2	0.010	0.02	0.025	0.02	0.05
≤ 0.0	C	≥ 6	0.02	0.04	0.2	0.005	0.01	0.02	0.02	0.04
≥ 0.5	C	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≥ 0.5	C	≥ 6	0.015	0.02	0.2	0.005	0.005	0.015	0.015	0.02
≤ 0.0	NC	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≤ 0.0	NC	≥ 6	0.01	0.015	0.2	0.0015	0.005	0.01	0.01	0.015
≥ 0.5	NC	≤ 3	0.01	0.015	0.2	0.005	0.01	0.01	0.01	0.015
≥ 0.5	NC	≥ 6	0.005	0.01	0.2	0.0015	0.005	0.005	0.005	0.01

FEMA-356

- If PR≤0.01 radians → PL = IO
- If 0.01<PR≤0.02 → PL = LS
- If 0.02<PR≤0.025 → PL = CP

No uncertainty in capacity

- PL = IO → p_{fPL} = P(PR=0.01)
- PL = LS → p_{fPL} = P(PR=0.02)
- PL = CP → p_{fPL} = P(PR=0.025)

Uncertainty in capacity

- PL = IO → $p_{fPL} \neq$ P(PR=0.01)
- PL = LS → $p_{fPL} \neq$ P(PR=0.02)
- PL = CP → $p_{fPL} \neq$ P(PR=0.025)

Reminder: **p**: probability, **P**: probability of exceedance (**POE**)

Conditional Probabilistic Approach: SAC/FEMA

Background

- Calculate the probability of exceedance (POE), **P**, of the i^th^ value of EDP

$$P\left(EDP^i\right)=\sum_m P\left(EDP^i \middle| IM_m\right) p\left(IM_m\right)$$

Computational model & simulations

Hazard curve

- Calculate the probability (**p**) of EDP^i^

$$\text{for } i = 1 : \#\text{ of EDP values}$$

$$p(EDP^i) = P(EDP^i) \quad \text{if } i = \#\text{ of EDP values}$$

$$p(EDP^i) = P(EDP^i) - P(EDP^{i+1}) \quad \text{otherwise}$$

From a previous slide

- Calculate the probability of not meeting a performance level (p_{fPL})

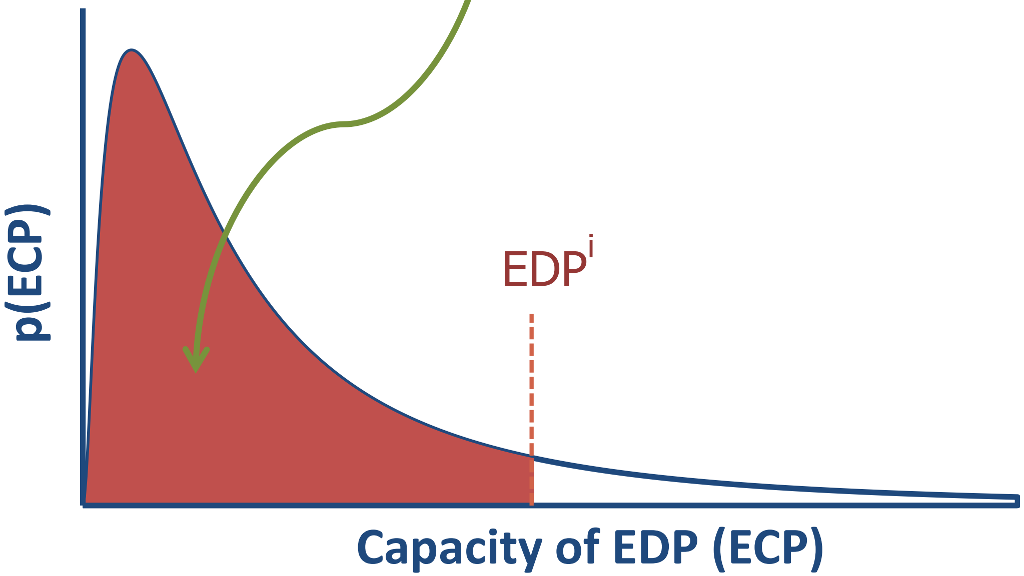
$$p_{fPL} = p[ECP \le EDP] = \sum_i p\left(ECP \le EDP^i \mid EDP^i\right) p\left(EDP^i\right)$$

Conditional Probabilistic Approach: SAC/FEMA

Background

- Calculate the probability of not meeting a performance level (p_{fPL})

$$p_{fPL} = p[\mathrm{ECP} \le \mathrm{EDP}] = \sum_i \boxed{p\left(\mathrm{ECP} \le \mathrm{EDP}^i \mid \mathrm{EDP}^i\right)}\, p\left(\mathrm{EDP}^i\right)$$



Conditional Probabilistic Approach: SAC/FEMA

Application Formats

- Approach requires large number of numerical simulations
- Computational effort introduced by the probability equations
- Two theoretically equivalent (with some practical differences) formats to reduce the computational burden:
  - Mean Annual Frequency (MAF) Format: A simple, closed-form evaluation of seismic risk (involving hazard, exposure & vulnerability)
  - Demand and Capacity Factored Design (DCFD) Format: A check of whether the building satisfies the selected limit-state requirements

Conditional Probabilistic Approach: SAC/FEMA

Some Definitions (Components of the **Risk**)

Hazard: Possible future occurrence of natural or human-induced physical events that may have adverse effects on vulnerable & exposed elements (A component of risk & not risk itself).

Exposure: Inventory of elements in an area in which hazard events may occur. If population & economic resources are not located in (exposed to) potentially dangerous settings, no problem if disaster risk would exist. Exposure is a necessary, but not sufficient, determinant of risk. It is possible to be exposed but not vulnerable. To be vulnerable to an extreme event, it is necessary to also be exposed.

Vulnerability: Propensity of exposed elements, e.g. humans & assets, to suffer adverse effects when impacted by hazard events. It is related to predisposition, susceptibilities, fragilities, weaknesses, deficiencies, or lack of capacities that favor adverse effects on the exposed elements.

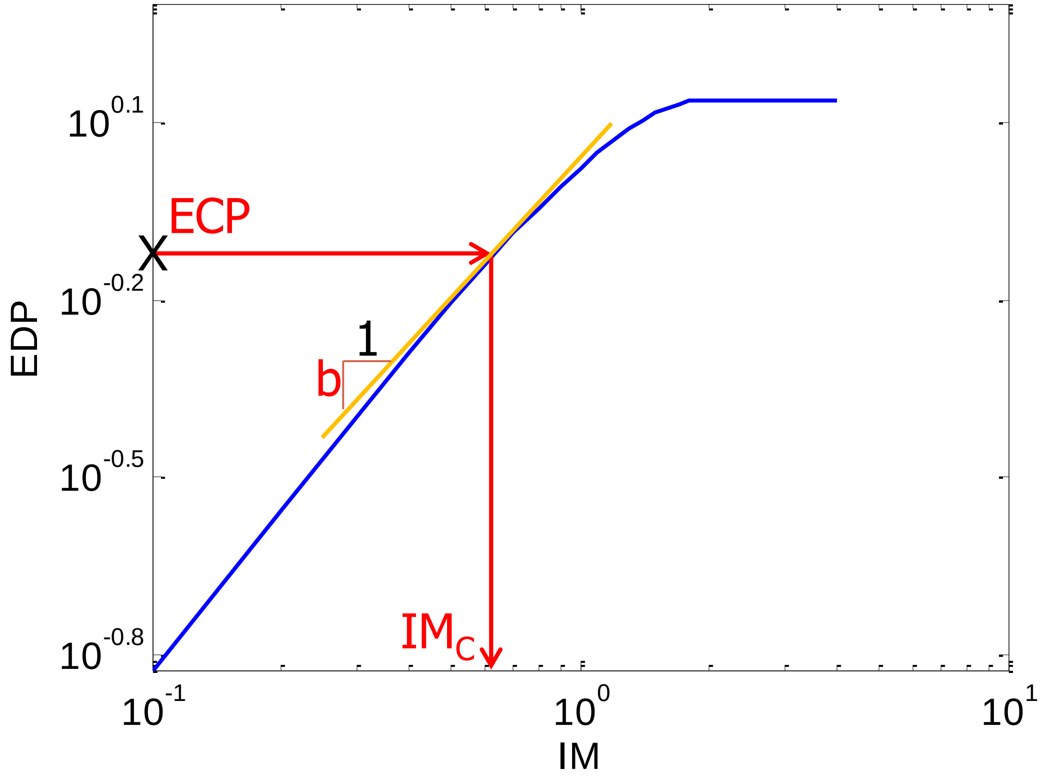
Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

IM_C: Value of IM that causes the structure to reach the EDP capacity (ECP) associated with the onset of the limit-state corresponding to the performance level PL.



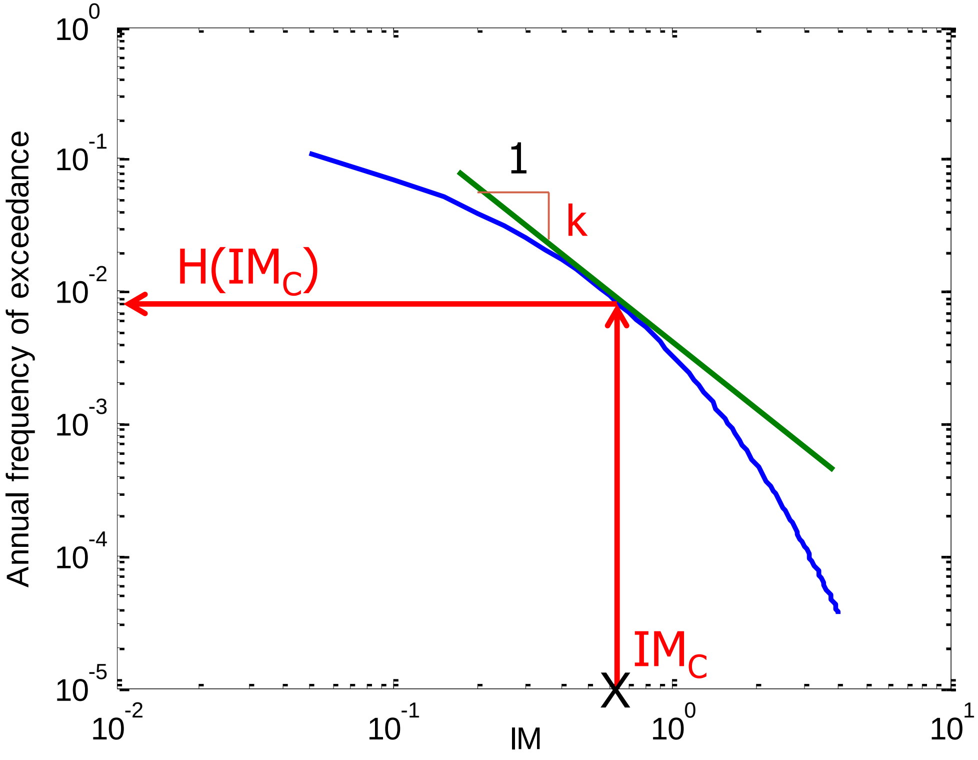
Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

$H(IM_C)$: Value of seismic hazard at IM_C



Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

Aleatory uncertainty (Randomness)

Epistemic Uncertainty

β_{DT}: Dispersion in Demand ⇒ $\beta_{DT} = \sqrt{\beta_{DR}^2 + \beta_{DU}^2}$

β_{CT}: Dispersion in Capacity ⇒ $\beta_{CT} = \sqrt{\beta_{CR}^2 + \beta_{CU}^2}$

Dispersion: Standard dev. of the log of the data

Aleatory Uncertainty

β_{DR}: Variability observed in structural response (Demand) from record-to-record
β_{CR}: Natural variability observed in tests to determine the EDP capacity (ECP) of a structural or non-structural component

Epistemic Uncertainty

β_{DU}: Uncertainty in modeling and analysis methods for estimating demand
β_{CU}: Incomplete knowledge of the structure for estimating capacity

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Mean annual frequency of not meeting a certain performance level PL: λ_{fPL}

$$\lambda_{fPL} = H(IM_C)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{DT}^2 + \beta_{CT}^2\right)\right]$$

Probability of not meeting a certain performance level PL: p_{fPL}

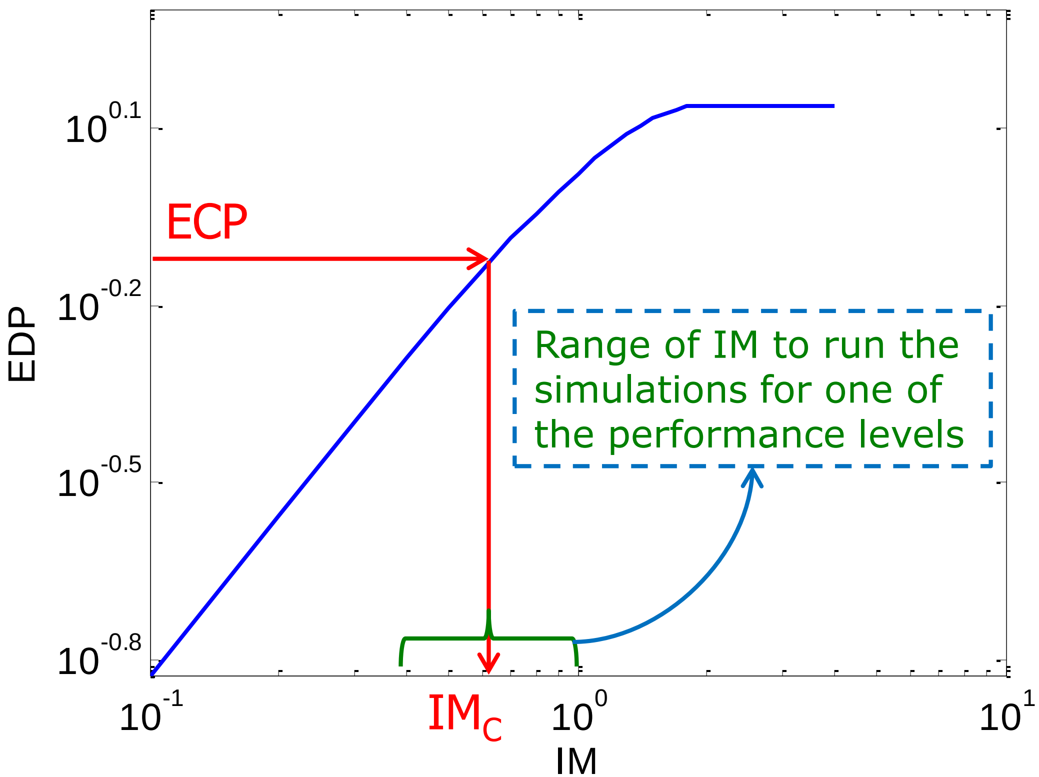
$$p_{fPL} = 1 - \exp\left(-\lambda_{fPL}\, t\right)$$ t: considered time period [years]

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: MAF

Advantage:

- Time history simulations do not need to be conducted for all IM values
- It may be sufficient to conduct the simulations for an estimated range of IM which covers the ECP values of the considered performance levels



Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

- A check of whether a certain performance level has been met or violated
- Resembles the familiar Load and Resistance Factor Design (**LRFD**) of modern design codes
- Unlike the MAF format, it cannot provide an estimate of the annual frequency of exceeding a given performance level

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

- FC: Factored capacity corresponding to the Performance Level
- FD_λ: Factored demand evaluated at the Hazard Level
- ECP_m: Median EDP capacity for the considered Performance Level
- $EDP_{m\lambda}$: Median demand evaluated at the IM level corresponding to λ

System Performance

Hazard Levels (Return Period)	Fully Operational	Operational	Life Safety	Near Collapse
Frequent (43 years)	●	o	o	o
Occasional (72 years)	Δ	●	o	o
Rare (475 years)	♦	Δ	●	o
Very rare (949 years)		♦	Δ	●

o: unacceptable performance

●: basic safety objective

Δ: essential hazardous objective

♦: safety critical objective

A performance objective:

Satisfy a Performance Level under a given Hazard Level

λ represents the annual frequency of exceedance associated with the Hazard Level

$$FC \geq FD_\lambda \Rightarrow \varphi \cdot ECP_m \geq \gamma \cdot EDP_{m\lambda},$$

$$\varphi = \text{Uncertainty Factor (\sim Strength Reduction Factor)},$$

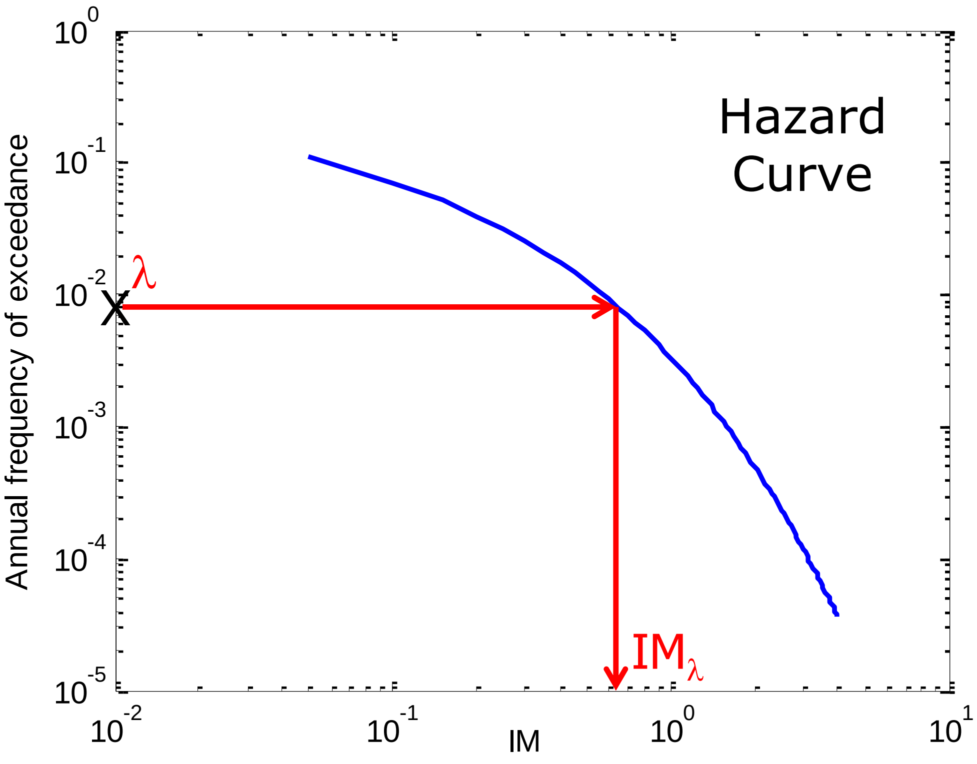
$$\gamma = \text{Uncertainty Factor (\sim Load Amplification Factor)}$$

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

$$FC \geq FD_{\lambda} \Rightarrow \varphi \cdot ECP_{m} \geq \gamma \cdot EDP_{m\lambda}$$

$EDP_{m\lambda}$: Median demand calculated at the IM value (IM_{λ}) corresponding to λ



Conditional Probabilistic Approach: SAC/FEMA

Application Formats: DCFD

$$\mathrm{FC} \geq \mathrm{FD}_\lambda \Rightarrow \varphi \cdot \mathrm{ECP}_m \geq \gamma \cdot \mathrm{EDP}_{m\lambda}$$

$$\varphi = \exp\left[-\frac{1}{2}\frac{k}{b}\left(\beta_{CR}^2 + \beta_{CU}^2\right)\right] \qquad \gamma = \exp\left[\frac{1}{2}\frac{k}{b}\left(\beta_{DR}^2 + \beta_{DU}^2\right)\right]$$

Remark:

- Median values are considered for capacity and demand
- Uncertainty is considered through the use of ϕ and γ
- Guarantees the Performance Objective with a confidence value greater than 50%
- Modifications have been made in DCFD to control and increase the confidence level: Enhanced DCFD (EDCFD)

Conditional Probabilistic Approach: SAC/FEMA

Application Formats: EDCFD

$$FC_R \geq FD_{R\lambda} \cdot \exp(K_x \beta_{TU}) \Rightarrow \varphi_R \cdot ECP_m \geq \gamma_R \cdot EDP_{m\lambda} \cdot \exp(K_x \beta_{TU})$$

$$\left.\begin{aligned} \varphi_R &= \exp\left[-\frac{1}{2}\frac{k}{b}\beta_{CR}^2\right] \\ \gamma_R &= \exp\left[\frac{1}{2}\frac{k}{b}\beta_{DR}^2\right] \end{aligned}\right\} \text{Only Aleatory uncertainty}$$

$$\left.\beta_{TU} = \sqrt{\beta_{DU}^2 + \beta_{CU}^2}\right\} \text{Epistemic uncertainty}$$

K_x: Standard normal variate (set of all random variables that obey a given probabilistic law) corresponding to the desired confidence level, α: $K_x = 1.28 \rightarrow \alpha = 90\%$; $K_x = 0.00 \rightarrow \alpha = 50\%$

EDCFD allows a user-defined **level of confidence** to be incorporated in the assessment.

Differing levels of confidence for:

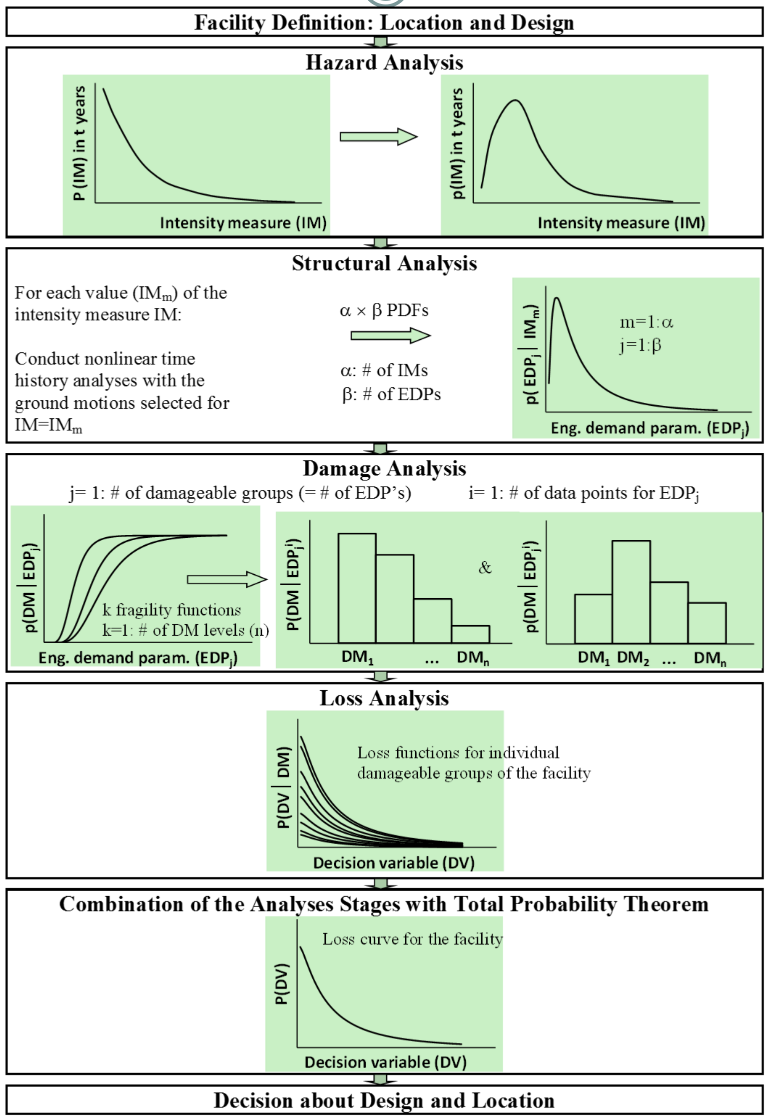
- Ductile versus brittle modes of failure (larger K_x for brittle)
- Local versus global collapse mechanisms (larger K_x for global)

Conditional Probabilistic Approach: PEER PBEE

- SAC/FEMA
  - Complete consideration of uncertainty & probability
  - Performance assessment not with decision variables (DV)
  - A special application of PEER PBEE framework

- PEER PBEE framework
  - Complete consideration of uncertainty & probability
  - Performance assessment with decision variables in terms of the direct interest of various stakeholders
  - Performance assessment considering:
    - Intensity Measure (IM)
    - Engineering Demand Parameter (EDP)
    - Damage Measure (DM)
    - Decision Variable (DV)

Conditional Probabilistic Approach: PEER PBEE



More about PEER PBEE in next part

Unconditional Probabilistic Approach: Introduction

Conditional Probabilistic Approach (CPA)

- Practice-oriented
- Conditioned on IM
- Obtain the p(IM) from hazard curve
- Employ recorded ground motions compatible with IM

Main difference

Unconditional Probabilistic Approach (UPA)

- More advanced
- Not conditioned on IM
- Stochastic models to directly describe the random time-series of seismic motion in terms of macro-seismic parameters, e.g. magnitude, distance, … etc.

- Synthetic ground motions are employed in UPA
- The main difference with the CPA is in the description of seismic motion at the site (**synthetic motions**)
- UPA-related research is mostly conducted up to generation of ground motions

Unconditional Probabilistic Approach: Introduction

- Methods of Unconditional Probabilistic Approach: Describe the randomness in the problem by a vector of random variables (**x**) where **x** should ideally cover the randomness in:
  - Earthquake source
  - Propagation path
  - Site geology/geotechnical aspects
  - Frequency content of the time-series
  - Structural response and capacity

- Simulations for **x** sampled from its probability distribution, f(**x**)

Unconditional Probabilistic Approach

Simulation Methods

Simulation:

- ❑ A robust way to explore the behavior of systems of any complexity
- ❑ Based on the observation of system response to input

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T \quad \Rightarrow \quad f(\mathbf{x}): \text{probability distribution for } \mathbf{x}$$

- ➢ Form a set of inputs of **x** from f(**x**) $\quad \mathbf{x}^{<i>} = \begin{bmatrix} x_{1i} & x_{2i} & \dots & x_{ni} \end{bmatrix}^T$
- ➢ Obtain the corresponding outputs
- ➢ Determine the distribution of the output through statistical post-processing

Unconditional Probabilistic Approach

Simulation Methods: Monte Carlo Simulation (MCS)

- A chosen set of inputs for **x**: $\mathbf{x}^{<i>} = \begin{bmatrix} x_{1i} & x_{2i} & \dots & x_{ni} \end{bmatrix}^T$
- If $\mathbf{x}^{<i>}$ fails in meeting certain performance requirements, then the contribution of $\mathbf{x}^{<i>}$ to the probability of not meeting those performance requirements (p_f) = $f(\mathbf{x}^{<i>})d\mathbf{x}$
- Then $p_f = \int_F f(\mathbf{x})d\mathbf{x}$

  F domain covers all $\mathbf{x}^{<i>}$ that fail in meeting the performance requirements

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int \underbrace{I_f(\mathbf{x})}_{\text{Indicator function}} f(\mathbf{x})d\mathbf{x} = E\left[I_f(\mathbf{x})\right]$$

$$\text{Indicator function} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ belongs to F} \\ 0 & \text{otherwise} \end{cases}$$

Unconditional Probabilistic Approach

Simulation Methods: Monte Carlo Simulation (MCS)

$$\boxed{p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int I_f(\mathbf{x})f(\mathbf{x})d\mathbf{x} = E\left[I_f(\mathbf{x})\right]}$$

Number of failed simulations

Monte Carlo Simulation: $p_f = E\left[I_f(\mathbf{x})\right] \cong \frac{1}{N}\sum_{i=1}^{N} I_f\left(\mathbf{x}^{<i>}\right) = \frac{N_f}{N} = \hat{p}_f$

Number of total simulations

- Obtain samples of $\mathbf{x}^{<i>}$ from the distribution $f(\mathbf{x})$
- Evaluate the performance of the structure for each $\mathbf{x}^{<i>}$
- Determine N_f and $\hat{p}_f$

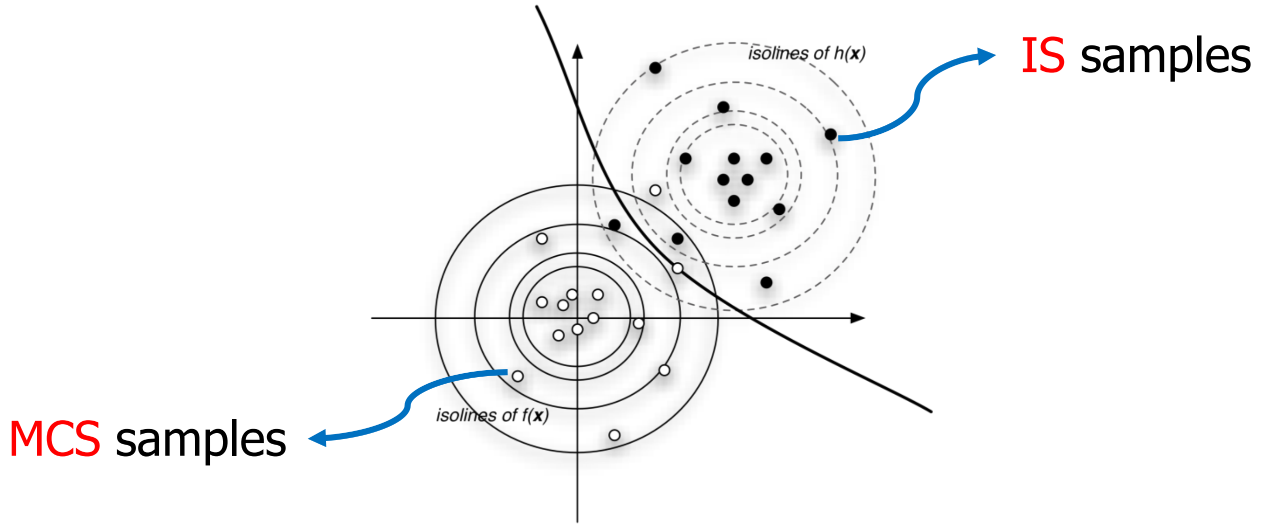
- $\hat{p}_f$ is an unbiased estimator of p_f
- Variance of $\hat{p}_f$ around p_f is proportional to p_f itself and decreases with increasing N

Unconditional Probabilistic Approach

Simulation Methods: Importance Sampling (IS)

- For very small values of p_f, N may need to be substantially large to obtain a few outcomes for N_f
- A possible solution to avoid excessive number of simulations $\rightarrow$

Importance sampling (IS): Sample according to a more favorable distribution



Unconditional Probabilistic Approach

Simulation Methods: Importance Sampling (IS)

Importance Sampling: The different ways of sampling must be accounted for

$$p_f = \int_F f(\mathbf{x})d\mathbf{x} = \int I_f(\mathbf{x})\boxed{\frac{f(\mathbf{x})}{h(\mathbf{x})}}h(\mathbf{x})d\mathbf{x} = E_h\left[I_f(\mathbf{x})\frac{f(\mathbf{x})}{h(\mathbf{x})}\right] \cong \frac{1}{N}\sum_{i=1}^{N} I_f\left(\mathbf{x}^{<i>}\right)\frac{f(\mathbf{x}^{<i>})}{h(\mathbf{x}^{<i>})}$$

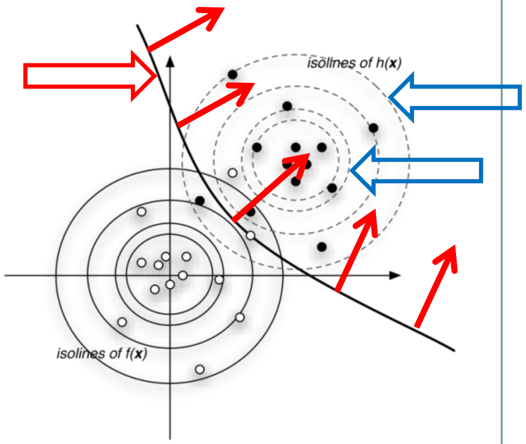
$$= \alpha(\mathbf{x})$$

IS weight

Sampling density

Requires some knowledge of the failure domain F

Requires a good sampling density h($\mathbf{x}$)



Unconditional Probabilistic Approach

Simulation Methods: IS w/ K-means Clustering (IS-K) (Jayaram & Baker, 2010)

- For both MCS & IS methods, some of the samples could be redundant
- IS-K method identifies & combines redundant samples →
- Reduces the number of simulations further

In its simplest version, IS-K consists of **five** main steps:

Step 1: Pick (randomly) K samples

Step 2: Calculate the cluster centroids (typically mean of the K samples)

Step 3: Assign each sample to the cluster with the closest centroid

Step 4: Recalculate the centroid of each cluster after the assignments

Step 5: Repeat steps 1 to 3 until no more reassignments (in step 4) take place

Once all the events are clustered, a single random sample from each cluster is used to represent all samples in that cluster

Unconditional Probabilistic Approach

Methodology for Seismic Assessment

Vector of random variables **x**:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \mathrm{M} \\ \mathrm{Z} \\ \mathrm{D} \\ \mathrm{w} \\ \mathrm{SR} \\ \mathrm{STR} \end{bmatrix}$$

M $\rightarrow$	Event magnitude M	Seismicity model
Z $\rightarrow$	Active fault/zone	
D $\rightarrow$	Source-to-site distance	
w $\rightarrow$	Stationary white noise	Ground motion model
SR $\rightarrow$	Soil	Site response model
STR $\rightarrow$	Structure	Structural model

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Seismicity Model

Seismicity model parameters M, Z and D sampled using simulation

Sampling for M

Monte Carlo Simulation

$$f(m) = \frac{\sum_{i=1}^{n_f} \lambda_i f_i(m)}{\sum_{i=1}^{n_f} \lambda_i}$$

$f_i(m)$: probability distribution of M for the i^{th} fault/source

λ_i: activation frequency for the i^{th} fault/source

(mean annual rate of all events on the source, i.e. events with M>Lower bound M for that source)

n_f: # active faults/sources

Importance Sampling

$$h(m) = \frac{1}{n_m} \frac{f(m)}{\int_{m_k}^{m_{k+1}} f(m)\,dm}$$

h(m): Sampling density for m lying in the k^{th} partition

n_m: # magnitude intervals (partitions) from m_{min} to m_{max}

Importance Sampling and K-means clustering

K-means clustering groups a set of observations into *K* clusters such that the dissimilarity between the observations within a cluster is minimized

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Seismicity Model

Seismicity model parameters M, Z and D sampled using simulation

Sampling for Z

Given that an earthquake with magnitude M = m has occurred, the probability that the event was generated in the i^{th} source is:

$$p(i|M=m)=\frac{\lambda_i f_i(m)}{\sum_{j=1}^{n_f}\lambda_j f_j(m)}$$

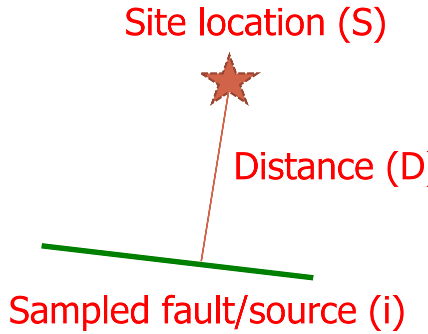
$f_i(m)$: probability distribution of M for the i^{th} fault/source
λ_i: activation frequency for the i^{th} fault/source
n_f: number of active faults/sources

Active zone (Z) is sampled from its discrete probability distribution conditioned on M

Sampling for D

There is no further effort needed to sample D.
It can be determined based on:

- The sampled fault/source
- The deterministic site location (S)



Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Synthetic Ground Motion Models:

- Seismologically-based Models
- Empirical Models

Seismologically-based Models

- Models that are based on the physical processes of earthquake generation and propagation
- Such models have reached a stage of maturity
- Applied in regions of the world where data is not sufficient for a statistical approach to seismic hazard
- Applied also in some regions of the world where seismic activity is well-known to (1) check their validity & (2) supplement existing information

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

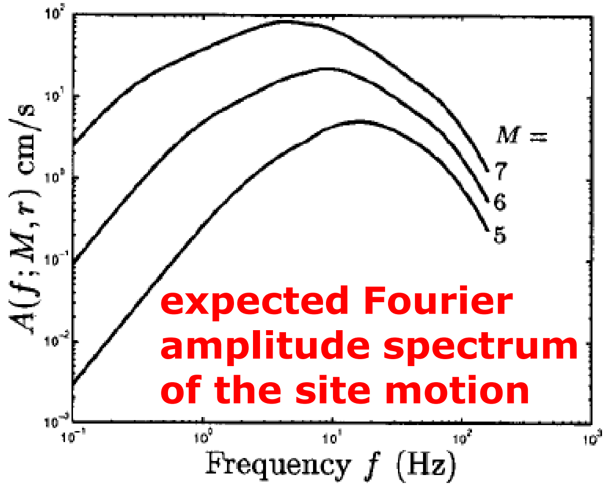
Seismologically-based Models (Atkinson & Silva, 2000)

- Acceleration-amplitude Fourier spectrum (or Radiation spectrum)
- Generation of time history

Acceleration-amplitude Fourier spectrum (Au & Beck, 2003, Pinto et al, 2004)

$$A(\mathbf{f},\mathbf{M},\mathbf{R}) = A_0(f)\frac{1}{R'}\exp(-\gamma(f)R')\cdot\exp(-\pi f\kappa)V(f) \;\&\; A_0(f) = CM_0(2\pi f)^2\left[\frac{1-\varepsilon}{1+(f/f_a)^2}+\frac{\varepsilon}{1+(f/f_b)^2}\right]$$

Source spectrum



Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Acceleration-amplitude Fourier spectrum (Au & Beck, 2003, Pinto et al, 2004)

near-surface attenuation: κ = 0.03

$$A(\mathbf{f},\mathbf{M},\mathbf{R}) = \boxed{A_0(f)}\frac{1}{R'}\exp(-\gamma(f)R')\cdot\exp(-\pi f\kappa)V(f)$$

Anelastic attenuation

Geometric spreading factor for direct waves

$$A_0(f) = CM_0(2\pi f)^2\left[\frac{1-\varepsilon}{1+(f/f_a)^2}+\frac{\varepsilon}{1+(f/f_b)^2}\right]$$

Source spectrum

$f_a=10^{2.18-0.496M}$, $f_b=10^{2.41-0.408M}$ Corner frequencies

$M_0=10^{1.5(M+10.7)}$ Seismic moment

$C=C_RC_PC_{FS}/(4\pi\rho\beta^3)$

$C_R=0.55$ Average radiation pattern for shear waves

$C_P=2^{-0.5}$ Accounts for partition of waves in two horizontal components

$C_{FS}=2$ Free-surface amplification

ρ & β Density & shear-wave velocity in the vicinity of the source

$\varepsilon=10^{0.605-0.255M}$ Corner frequencies weighted through this parameter

$R' = \sqrt{h^2+R^2}$ Radial distance between source and site

R Epicentral distance

$h=10^{-0.05+0.15M}$ Nominal depth of fault [km] ranging from ~ 5 km for M=5 to 14 km for M=8

$\gamma(f)=\pi f/(Q\,\beta)$, $Q=180f^{\,0.45}$ Regional quality factor

$V(f)$ *Describes the amplification* through the crustal velocity gradient (wave passage)

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Seismologically-based Models

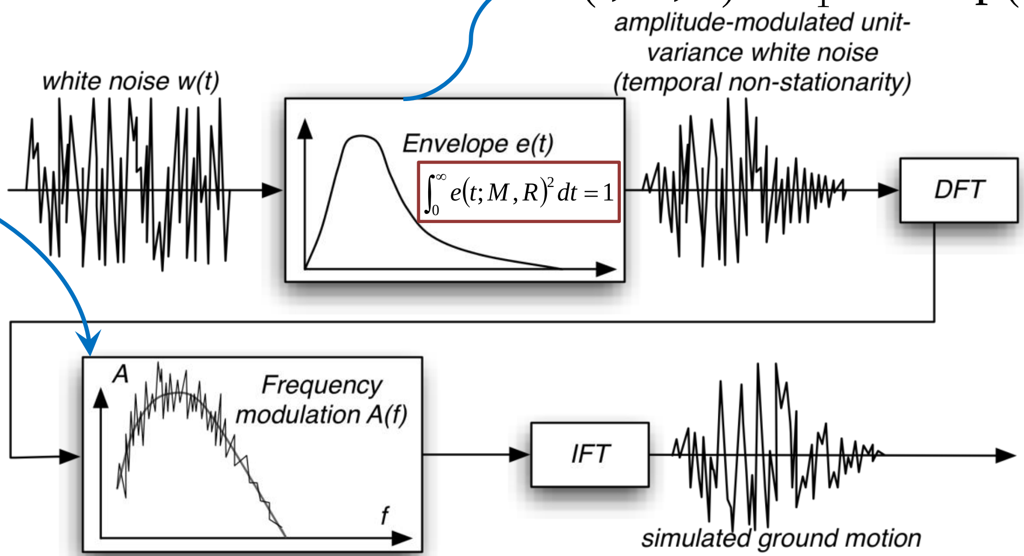
- Acceleration-amplitude Fourier spectrum (or Radiation spectrum)
- Generation of time history

Dependence on *M* & *R* introduced through α_3
α_1: Normalizing factor → envelope has **unit energy**
U(*t*): Unit-step function

Generation of time history

multiply

$$e(t;M,R) = \alpha_1 t^{\alpha_2 - 1} \exp(-\alpha_3 t) U(t)$$



DFT: Discrete Fourier Transform

IFT: Inverse Fourier Transform

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Synthetic Ground Motion Models:

- Seismologically-based Models
- Empirical Models

Empirical Models

- Models consist of parameterized stochastic (random) process models
- Developed by observing that ground motions possess stable statistical nature given earthquake and site characteristics (M, R & soil type)
- This observation led to the idea of considering the ground motion acceleration time-series as samples of random processes

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Ground Motion Model

Synthetic Ground Motion Models:

- Seismologically-based Models
- Empirical Models

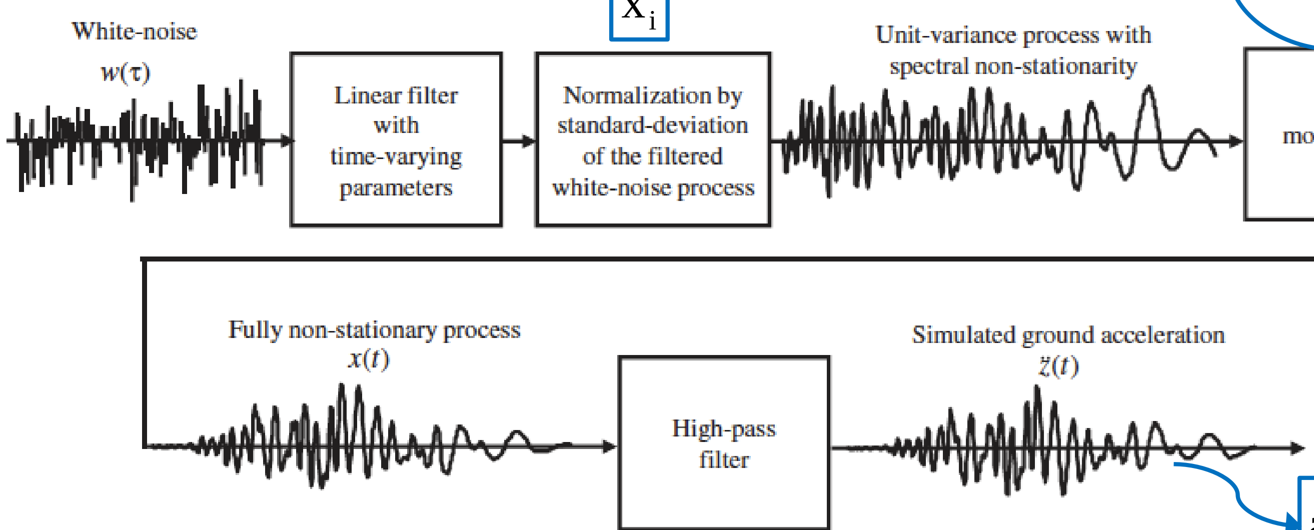
$$\omega_g = \omega_g(t)$$
$$\xi_g = \xi_g(t)$$

Empirical Models (Rezaeian & Der Kiureghian, 2010)

The filter **IRF** (Impulse Response Function) is the acceleration IRF of a linear SDOF oscillator of natural frequency ω_g and damping ratio ξ_g

$$h_i(t) = h(t-\tau_i) = \frac{\omega_g}{\sqrt{1-\xi_g^2}} \exp\left[-\xi_g \omega_g (t-\tau_i)\right] \sin\left[\omega_g \sqrt{1-\xi_g^2}\,(t-\tau_i)\right]$$

x_i



$$a(t) = \sum_{i=1}^{n} x_i h_i(t)$$

Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Site-Response Model

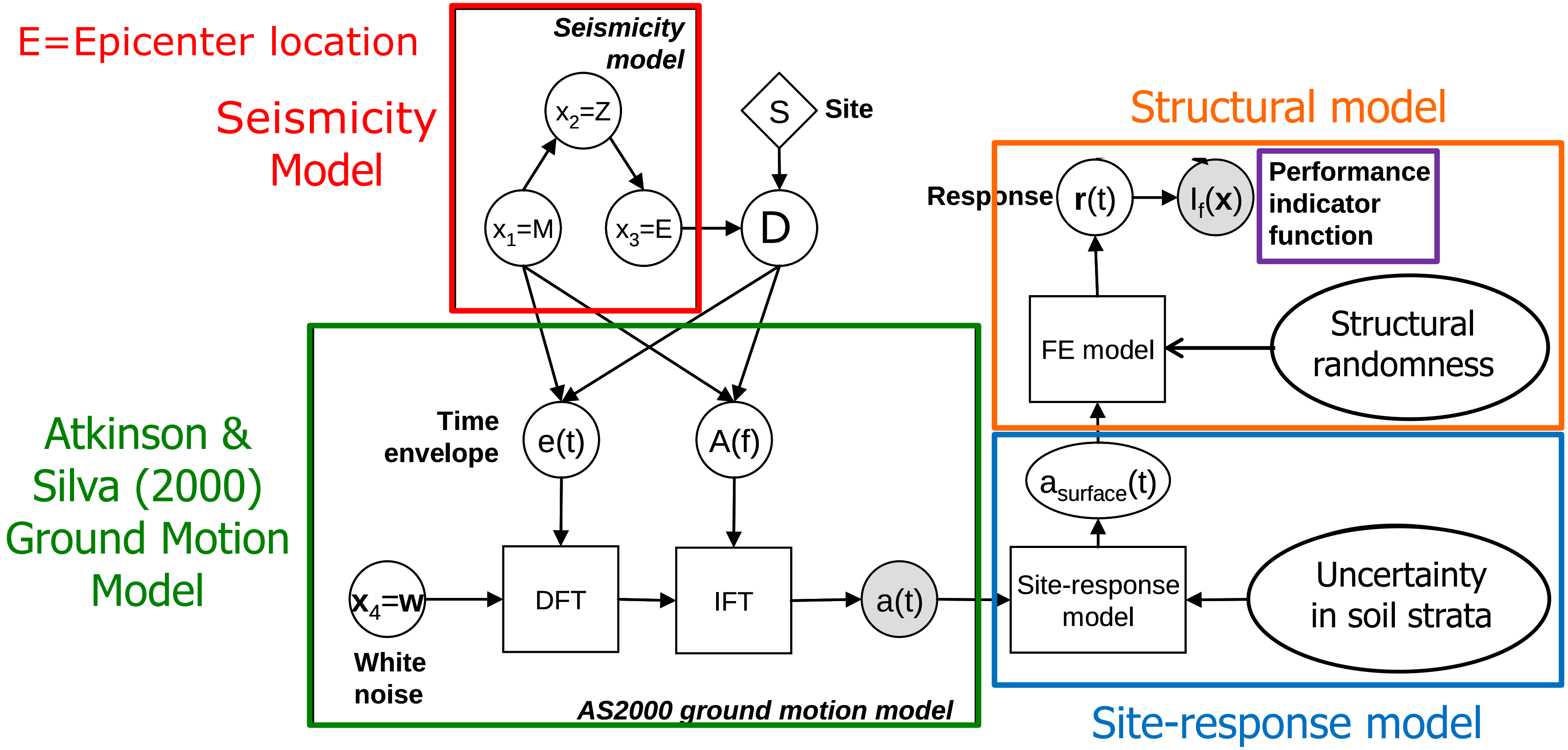
- Ground motion model determines ground motion time history for bedrock
- A site response model is used to obtain input motion to the structure at the surface
- Model the soil strata and corresponding stiffness, strength & damping properties, e.g. a one-dimensional nonlinear, or equivalent linear, model
- The strata thicknesses and properties possess uncertainty

Methodology for Seismic Assessment: Structural Model

- Finite element model which determines the response of the structure
- Both the structure itself, and the response-model implemented in the analysis software, are affected by uncertainty (more later!)

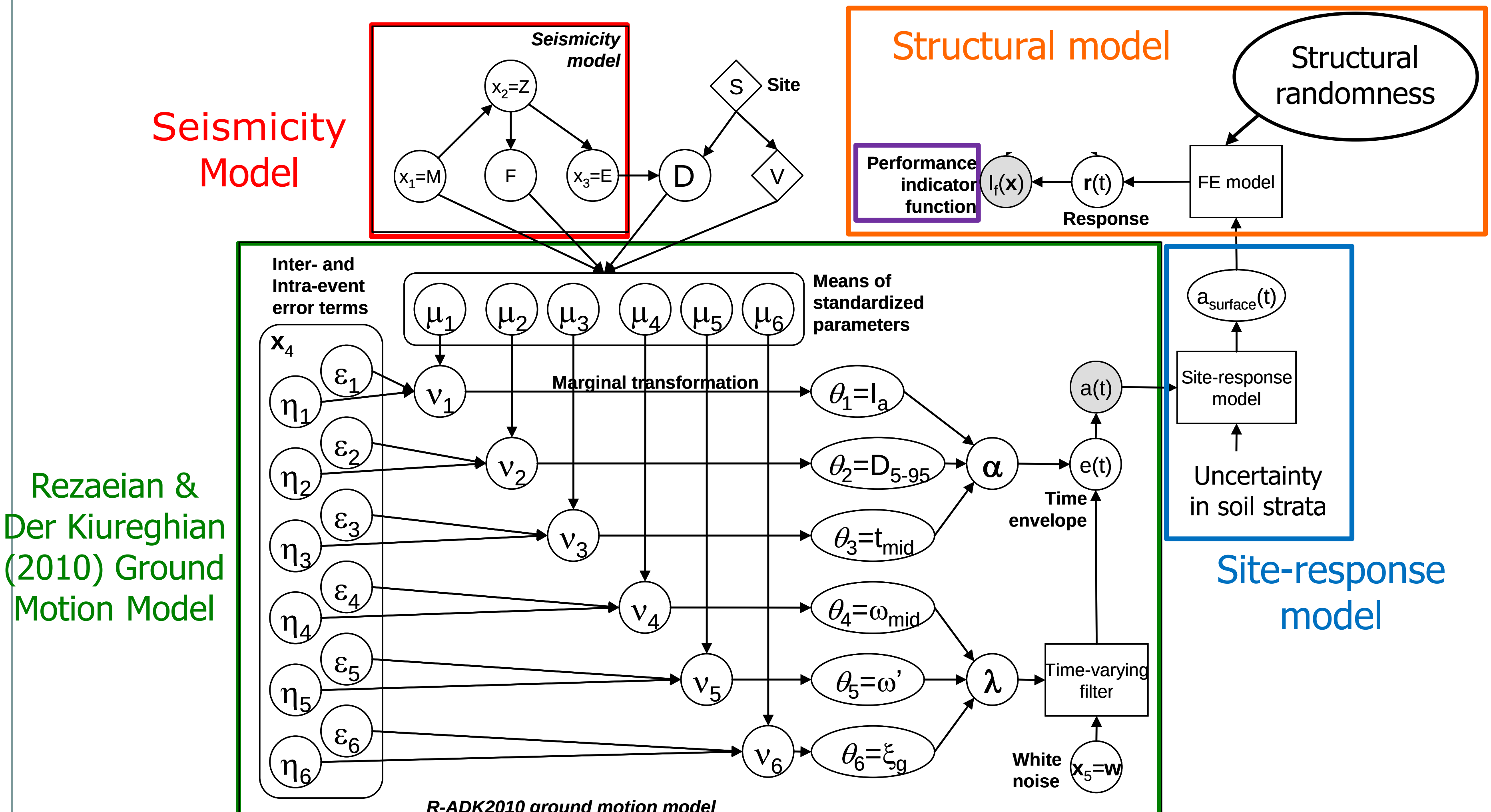
Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Flowchart



Unconditional Probabilistic Approach

Methodology for Seismic Assessment: Flowchart



Unconditional Probabilistic Approach

Methodology for Seismic Assessment:

Application to the Estimation of a Structural Mean Annual Frequency

POE of PL given an event occurs from the i^{th} source

$$\sum_{i=1}^{N}\lambda_i$$

MAF of a performance level (PL)

$$\lambda_{PL} = \sum_{i=1}^{N}\lambda_i P_{PL|i} = \lambda_0\sum_{i=1}^{N}(\lambda_i/\lambda_0)P_{PL|i} = \lambda_0\sum_{i=1}^{N}P_{PL|i}\cdot p_i = \lambda_0 P_{PL}$$

i^{th} seismic source

POE of PL: could be calculated with the seismic assessment flowchart (**2 previous slides**)

λ_i: activation frequency for the i^{th} fault/source

(mean annual rate of all events on the source, i.e. events with M>Lower bound M for that source)

Questions?

mosalam@berkeley.edu

http://www.ce.berkeley.edu/people/faculty/mosalam

I-2 PBEE Design Methods

KHALID M. MOSALAM, PROFESSOR

UNIVERSITY OF CALIFORNIA, BERKELEY

Outline

1. **Introduction**
2. **Optimization-based methods**
3. **Non optimization-based methods**

Introduction

Courtesy of Prof. S. Mahin

Robust structures & systems needed to account for extensive variability in earthquake & structural characteristics

Introduction

- Performance of a structure under earthquake excitation depends on:
  - Earthquake characteristics
  - Proximity to fault rupture
  - Soil and foundation type
  - Structural system
  - Configuration and details
  - Nonstructural components
  - Quality of engineering
  - Quality of construction
- Probabilistic seismic design is the direct design method which considers the uncertainty and variability of the above items
- The state of development of fully probabilistic seismic design methods is behind that of assessment methods

Optimization-based methods

Structural optimization problems can be expressed as:

Vector of decision variables

$$\min \; f(\mathbf{x}) \;\text{ subject to }\; g(\mathbf{x} \le \alpha)$$

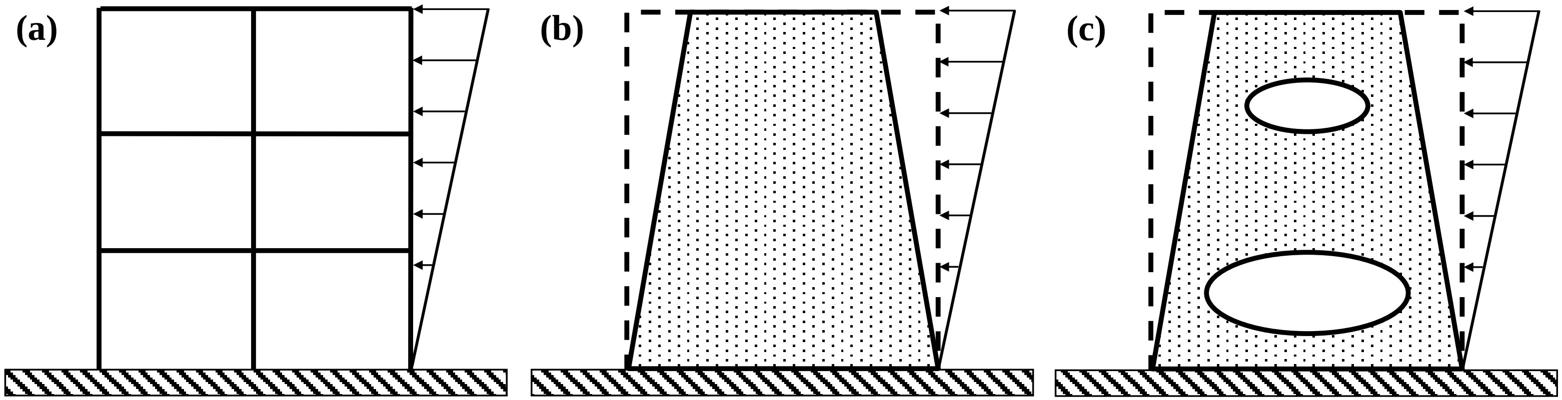
Objectives Constraints

Although this is a common notation for almost all optimization problems, the structure being optimized, variables, constraints and the domain of optimization can be significantly different.

Optimization-based methods

Classification

Sizing | Shape | Topology



Sizing

- Both the size and location of structural members are varied
- Dimensions are varied to obtain the optimal solutions

Shape

- Connectivity of the structure is fixed
- Shape (boundary) is varied to obtain the optimal solution

Topology

- Locations and number of elements are fixed and known
- Formation of new boundaries is allowed

Most optimization studies on structural earthquake engineering deal with sizing, where the design variables are limited to member/section properties

Optimization-based methods

Terminology

- Objective (merit) function: A function that measures the performance of a design
  - Takes a different value for every design alternative
  - **Ex.**: Maximum interstory drift ratio (MIDR), initial cost, …

- Design (decision) variables: A vector (of size k) that defines the design
  - Each element in the vector describes a different structural property relevant to the optimization problem
  - Take different values throughout the optimization process
  - **Ex.**: Section dimensions, reinforcement ratios, …

- Constraint: A condition that a solution of the optimization problem should satisfy
  - **Ex.**: Traditional code design requirements

Optimization-based methods

Terminology

- Space of design (decision) variables (search space): Space defined by the range of design (decision) variables
  - k dimensions: k is the number of design variables in the problem
  - Each dimension: either continuous or discrete depending on the nature of the corresponding design variable

- Solution (objective function) space: Space defined by the objective function
  - Usually the solution space is unbounded or semi-bounded
  - n dimensions: n is the number of objective functions in the problem
  - The optimal solutions are defined in the solution space
  - The set of optimal solutions in the solution space is referred to as a Pareto-front or Pareto-optimal set

Vilfredo Pareto (1848–1923): Italian economist

Optimization-based methods

Terminology

- Pareto-optimality:

Minimize $f(\mathbf{x})$

the objective function

$\mathbf{y}^{<i>} = f\left(\mathbf{x}^{<i>}\right)$

A point in the search space

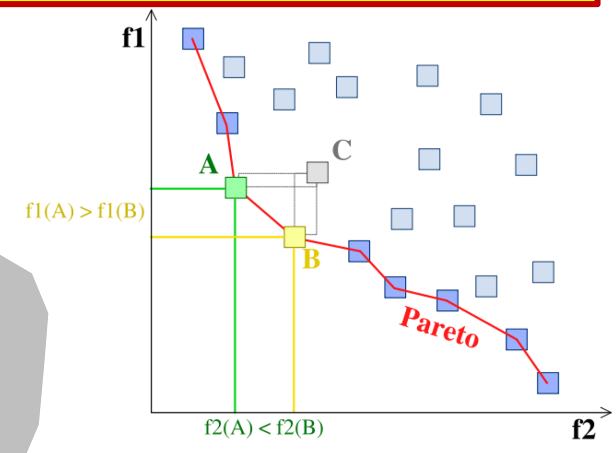
Corresponding point in the solution space

$\mathbf{y}^{<i>}$ dominates $\mathbf{y}^{<j>}$ *if* $\mathbf{y}_n^{<i>} < \mathbf{y}_n^{<j>}$ for all n

$\mathbf{y}^{<i>}$ strictly dominates $\mathbf{y}^{<j>}$ *if* $\mathbf{y}_n^{<i>} < \mathbf{y}_n^{<j>}$ for at least one n

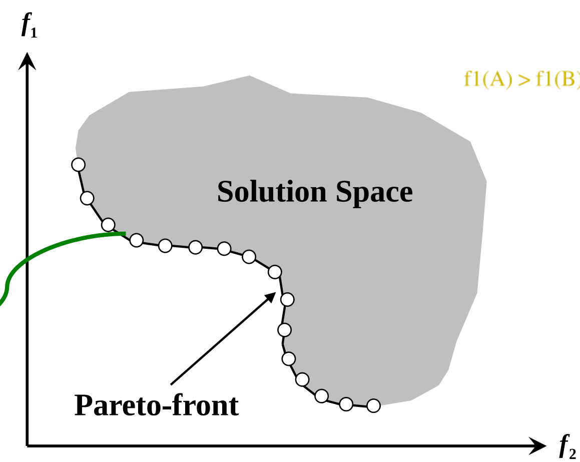
Recall: n is the number of objective functions in the problem

Boxed points are feasible choices with preferred smaller values. *C* is not on the Pareto Front because it is dominated by *A* & *B*. *A* & *B* are not strictly dominated by any others, and hence lie on the front.



Pareto-front (Y*) is the subset of points that are not strictly dominated by another point.

Boundary that minimizes objectives f_1 and f_2



Optimization-based methods

Terminology

- <u>Performance levels</u>: Levels that describe the performance of the structure against earthquake hazard
  - Exceedance of each performance level is determined based on the crossing of a threshold value (with a probabilistic distribution) in terms of structural capacity
  - **Ex.**: Immediate Occupancy (IO), Life Safety (LS), Collapse Prevention (CP)

- <u>Hazard levels</u>: Probability levels used to describe the earthquake intensity
  - Usually defined in terms of earthquake mean return periods or probability of exceedance (POE) during a certain duration
  - **Ex.**: 2475 years (2% POE in 50 years), 72 years (50% POE in 50 years)

- <u>Performance objective</u>: Achieving a Performance Level under a given Hazard Level

Optimization-based methods

Tools: Earlier Studies

- Focused on single-objective optimization using gradient-based algorithms
  - These algorithms aim to minimize or maximize a real function by systematically choosing variables from within an allowed search space
  - Most commonly used types: linear and nonlinear programming, optimality criteria, and feasible directions
  - Computationally efficient due to rapid convergence rates
  - Require the existence of continuous objective functions and constraints in order to evaluate gradients, so the range of application is limited
- Objective function was almost exclusively selected as the initial cost or the material usage
- Several constraints (most often based on code provisions) were applied to determine the validity of designs
- Explicit formulations, which could be evaluated with little effort, were used for both the objective function and the constraints

Optimization-based methods

Tools: Modern Studies

- Most practical design problems in structural engineering require discrete representation of design variables (e.g. section sizes, reinforcement areas, …)
- The advent of numerical structural analysis methods has led to objective functions and/or constraints that are naturally discontinuous (e.g. EDPs)
- Researchers resorted to zero-order optimization algorithms that do not require existence of gradients or continuity of objective functions or constraints
- A class of zero-order optimization algorithms is the heuristic methods:
  - Genetic algorithms (GA)
  - Simulated annealing (SA)
  - Tabu search (TS)
  - Shuffled complex evolution (SCE)

Optimization-based methods

Tools: Modern Studies

Advantages of the heuristic methods:

- Can be adapted to solve any optimization problem with no requirements on the objectives and constraints
- Very effective in finding the global minimum of highly nonlinear and/or discontinuous problems whereas gradient-based algorithms can easily be trapped at a local minimum

Criticism of the heuristic methods:

- Experience-based and depend on an improved version of basic trial-and-error
- Not based on a mathematical theory and there is no single heuristic optimization algorithm that is general for a wide class of optimization problems

Optimization-based methods

Tools: Modern Studies

Tabu Search (Glover, 1989, 1990)

- Generally, used to solve combinatorial optimization problems (i.e. a problem of finding an optimum solution within a finite set of feasible solutions)
- Employs a neighborhood search procedure to sequentially move

  From a combination of design variables $\mathbf{x}^{<i>}$, e.g. section sizes, reinforcement ratios, …, having a unique solution $\mathbf{y}^{<i>}$, e.g. MIDR, life cycle cost (LCC), …

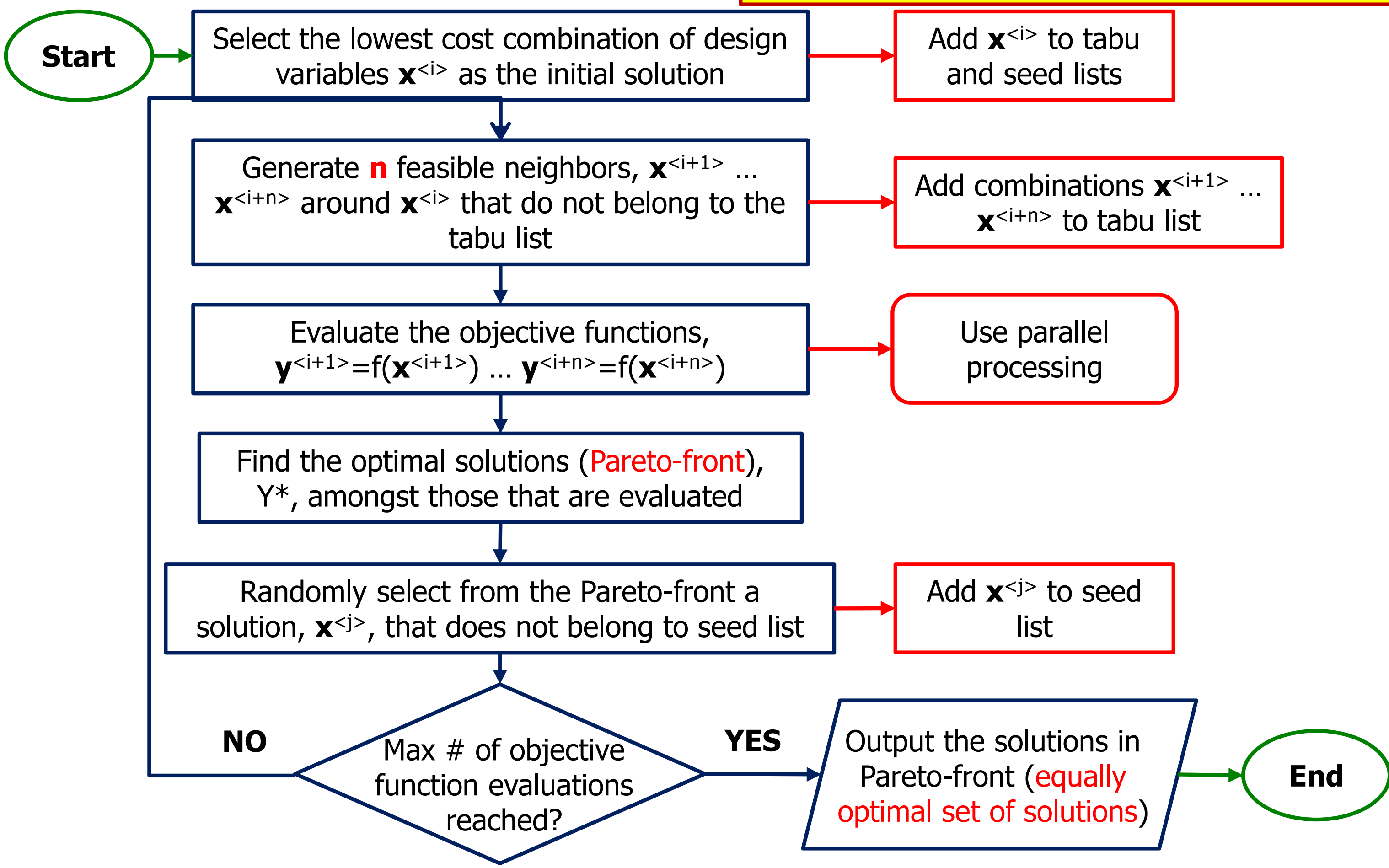
  To another combination in the neighborhood of $\mathbf{x}^{<i>}$ until some termination criterion has been reached ($\mathbf{x}^{<i>}$: seed point)
- Usually a portion of the neighboring points is selected randomly to prevent the algorithm to be trapped at a local minimum
- Keeps track of all previously employed $\mathbf{x}^{<i>}$ (tabu list & seed list), which are excluded from the set of neighboring points that are determined at each iteration
- Naturally lends itself to parallel processing, often needed to solve problems when evaluating the objective functions or the constraints is computationally costly

Optimization-based methods

Tools: Modern Studies

Tabu Search (Glover, 1989, 1990)

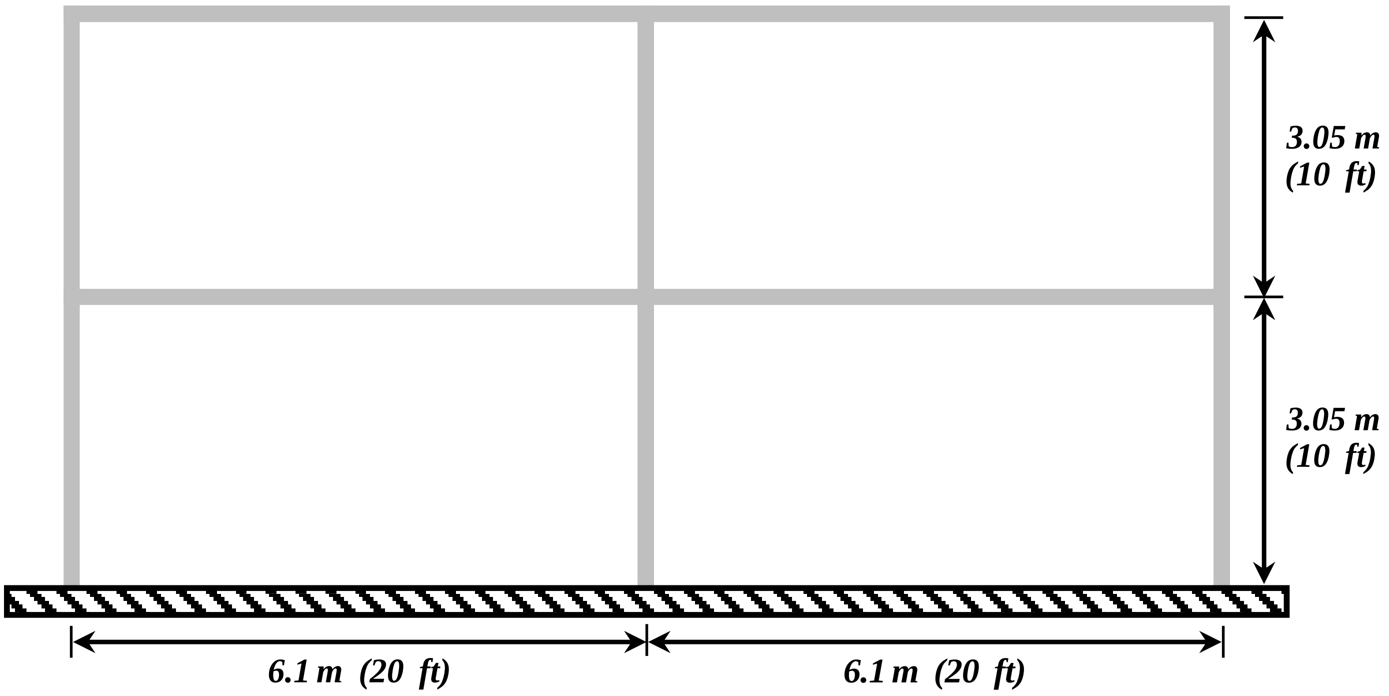
Enhances local search by relaxing its basic rule. ***Worsening*** moves can be accepted if no improving move is available, e.g. search is stuck at strict local minima. ***Prohibitions*** (henceforth the term ***tabu***) are introduced to discourage search from coming back to previously-visited solutions.



Optimization-based methods

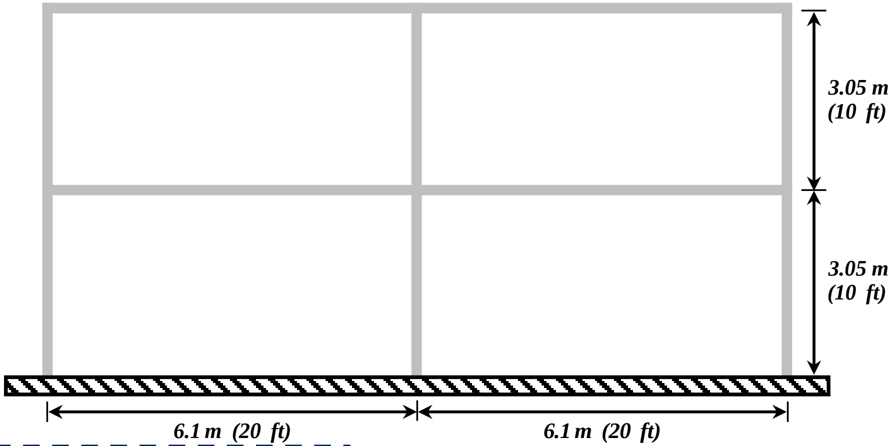
Illustrative Example:

Optimization of a two-story two-bay RC frame



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame



Design Variables (x):

- Column Reinforcement Ratio (%)
- Beam Reinforcement Ratio (%)
- Width of Exterior Columns (mm)
- Width of Interior Columns (mm)
- Depth of Columns (mm)
- Depth of Beams (mm)
- Width of Beams (mm)

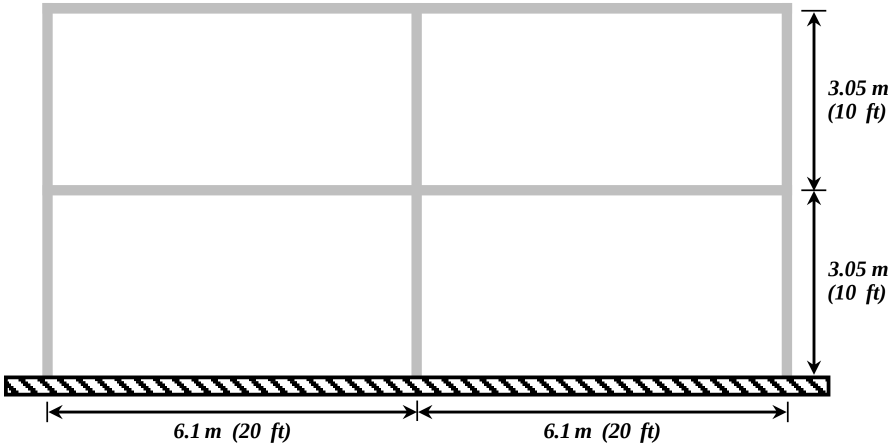
Objective Functions [y=f(x)]:

- Initial Cost
- Expected value of Life-cycle Cost (LCC)
- Maximum Interstory Drift Ratio (MIDR)

Target: Minimize the objective functions

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame



Search Space

Design Variables	Minimum	Maximum	Increment
Column Reinforcement Ratio (%)	1.0	3.0	0.5
Beam Reinforcement Ratio (%)	1.0	3.0	0.5
Width of Exterior Columns (mm)	304.8	508	50.8
Width of Interior Columns (mm)	355.6	558.8	50.8
Depth of Columns (mm)	304.8	457.2	50.8
Depth of Beams (mm)	406.4	558.8	50.8
Width of Beams (mm)	304.8	406.4	50.8

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

Objective Functions [$y=f(x)$]

- Initial Cost (C_0):
  - C_0 = Cost of (Steel + Concrete + Formwork + Labor)
  - Estimated according to 2011 Building Construction Cost Data
- Expected Value of Life Cycle Cost ($E[LCC]$):
  - LCC is a random quantity due to various sources of uncertainty including
    - Ground motion variability,
    - Modeling error (see next slide),
    - Unknown material properties
  - The expected LCC of a structure, incorporating both aleatory uncertainty due to ground motion variability and epistemic uncertainty due to modeling error, is expressed as follows:

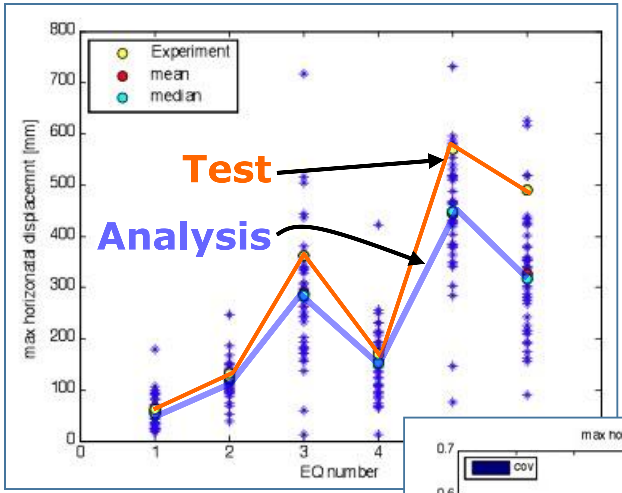
See Slide after next

$$E[LCC] = C_0 + \boxed{\int_0^L E[C_{SD}]\left(\frac{1}{1+\lambda}\right)^t dt} = C_0 + \boxed{\alpha L E[C_{SD}]}$$

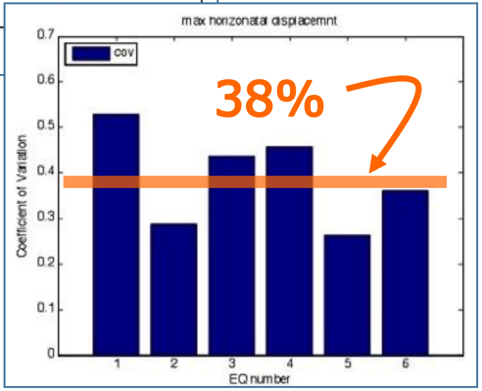
Optimization-based methods

Courtesy of Prof. S. Mahin

Modeling error



41 expert teams participated



Full-scale 1D tests of circular column - J. Restrepo, PI (PEER, Caltrans, UNR, FHWA, NEEScomm & NSF)

Gets more complicated in a building: Effect of finite joint sizes, gravity system, non-structural components (cladding, partitions, stairs, etc)

May lead to incorrect estimation of performance

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

- Expected Value of Life Cycle Cost (E[LCC]):

Expected seismic damage cost

(Assumed to be governed by a Poisson's process)

$$E[LCC] = C_0 + \int_0^L E[C_{SD}]\left(\frac{1}{1+\lambda}\right)^t dt$$

Poisson process: A stochastic process where time between pairs of consecutive events has exponential distribution & these inter-arrival times is assumed independent of other inter-arrival times.

$$E[LCC] = C_0 + \alpha \underbrace{L}_{\text{Life span}} \boxed{E[C_{SD}]} \qquad \alpha = [1 - \exp(-qL)]/qL \qquad q = \ln(1+\underbrace{\lambda}_{\text{Annual discount rate}})$$

$$E[C_{SD}] = \sum_{i=1}^{\boxed{N}} C_i p_i$$

Probability of i^{th} damage state:

$$p_i = p(\Delta_D > \Delta_{C,i}) - p(\Delta_D > \Delta_{C,i+1})$$

D: demand, C: capacity

Examples in SAC/FEMA

Cost for i^{th} damage state:

- 30% IO-LS
- 70% LS-CP
- 100% CP

N=Total number of considered damage-states:

- IO-LS (state between Immediate Occupancy & Life Safety)
- LS-CP (state between Life safety & Collapse Prevention)
- CP (Collapse Prevention)

Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

SAC/FEMA equation:

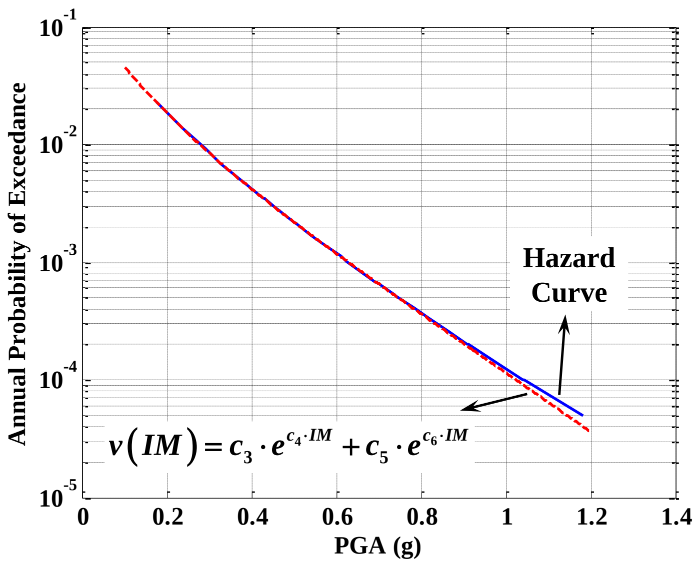
$$p\left(\Delta_D > \Delta_{C,i}\right) = \int_0^{\infty} \underbrace{p\left(\Delta_D > \Delta_{C,i} \mid IM = im\right)} \underbrace{\left|\frac{d\nu(IM)}{dIM}\right|} dIM$$

Conditional probability of demand being greater than the capacity given the ground motion intensity [**See next slide**]

Slope of the hazard curve: Possible to obtain analytically by fitting a function to the curve

$$\nu(IM) = c_3 \cdot \exp(c_4 \cdot IM) + c_5 \cdot \exp(c_6 \cdot IM)$$

$$\frac{d\nu(IM)}{dIM} = c_3 \cdot c_4 \cdot \exp(c_4 \cdot IM) + c_5 \cdot c_6 \cdot \exp(c_6 \cdot IM)$$



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

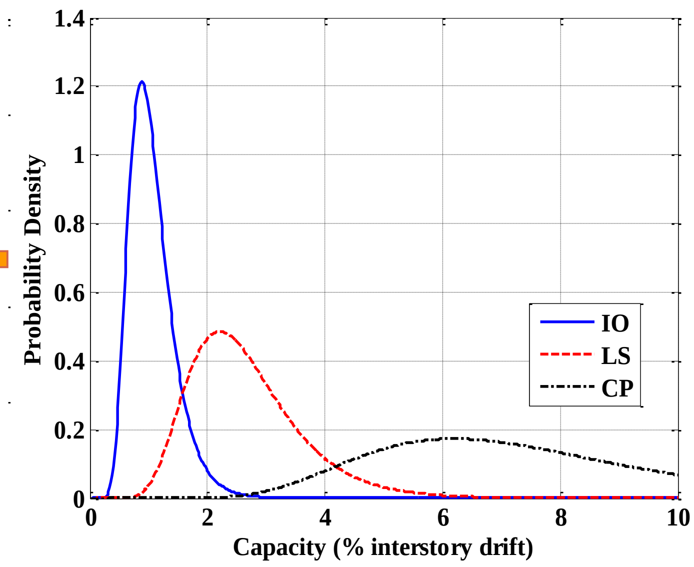
$$p\left(\Delta_D > \Delta_{C,i} \mid IM = im\right) = \int_0^{\infty} \underbrace{p\left(\Delta_D > \delta \mid IM = im\right)}_{\text{See next slide}} \boxed{f_{C,i}(\delta)}\, d\delta$$

See next slide

Probability density function for structural capacity for the ith damage state

Lognormal distribution with logarithmic mean & standard deviation $\Delta_{C,i}$ & β_C, respectively.

The uncertainty in capacity represented with β_C accounts for factors such as modeling errors & variations in material properties



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

$$p\left(\Delta_D > \delta \mid IM = im\right) = 1 - \Phi\left[\frac{\ln(\delta) - \lambda_{D|IM=im}}{\beta_D}\right]$$

mean of the natural logarithm of the earthquake demand (function of ground motion intensity) [$\lambda_D = \ln(\mu_D)$]

Logarithmic standard deviation of the corresponding normal distribution of the earthquake demand

Standard normal cumulative distribution

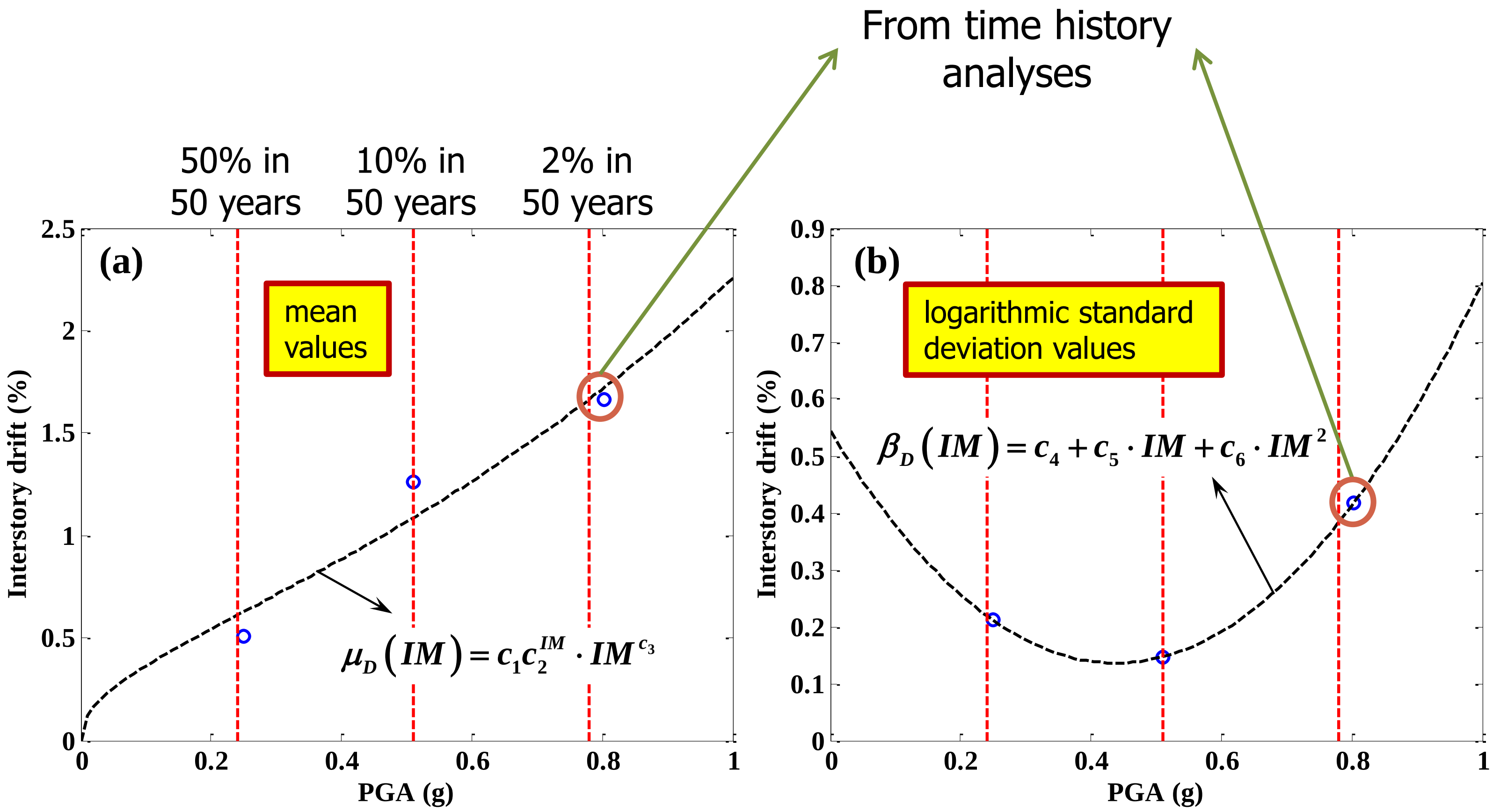
It is possible to describe μ_D & β_D as continuous functions of the ground motion intensity (Aslani and Miranda, 2005)

$$\mu_D(IM) = c_1 c_2^{IM} IM^{c_3} \qquad \beta_D(IM) = c_4 + c_5 \cdot IM + c_6 \cdot IM^2$$

Constants c_1–c_3 & c_4–c_6 are determined by fitting a curve to the mean & logarithmic standard deviation values obtained from time history analyses of the analytical model

Optimization-based methods

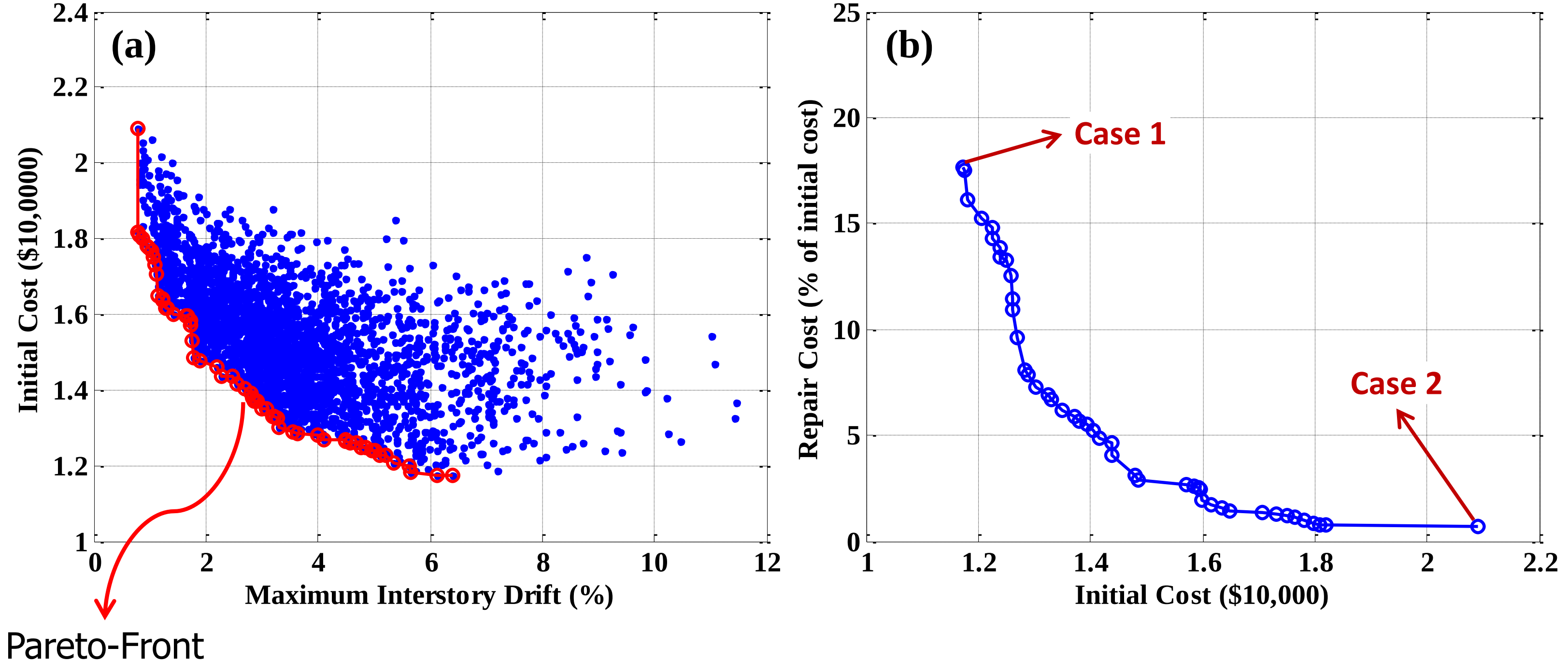
Illustrative Example: Optimization of a two-story two-bay RC frame



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

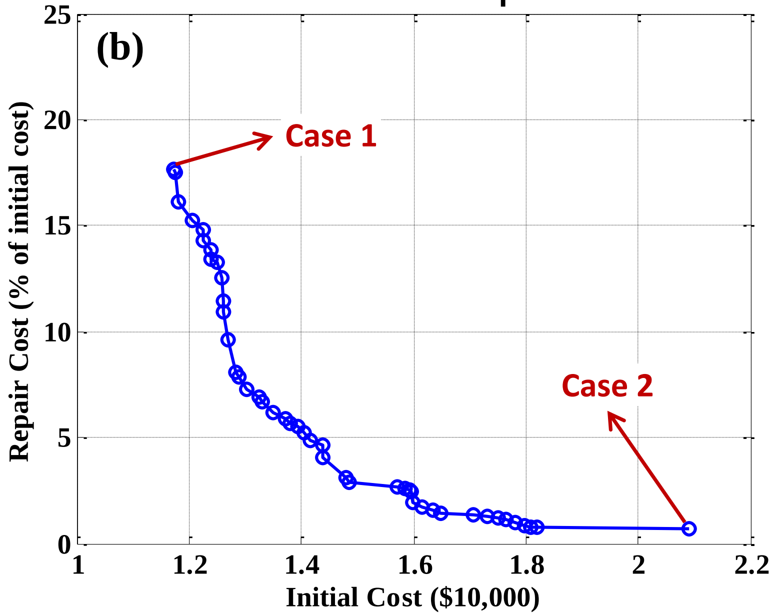
Optimization Results with Tabu Search (TS) algorithm



Optimization-based methods

Illustrative Example: Optimization of a two-story two-bay RC frame

Optimization Results with TS algorithm



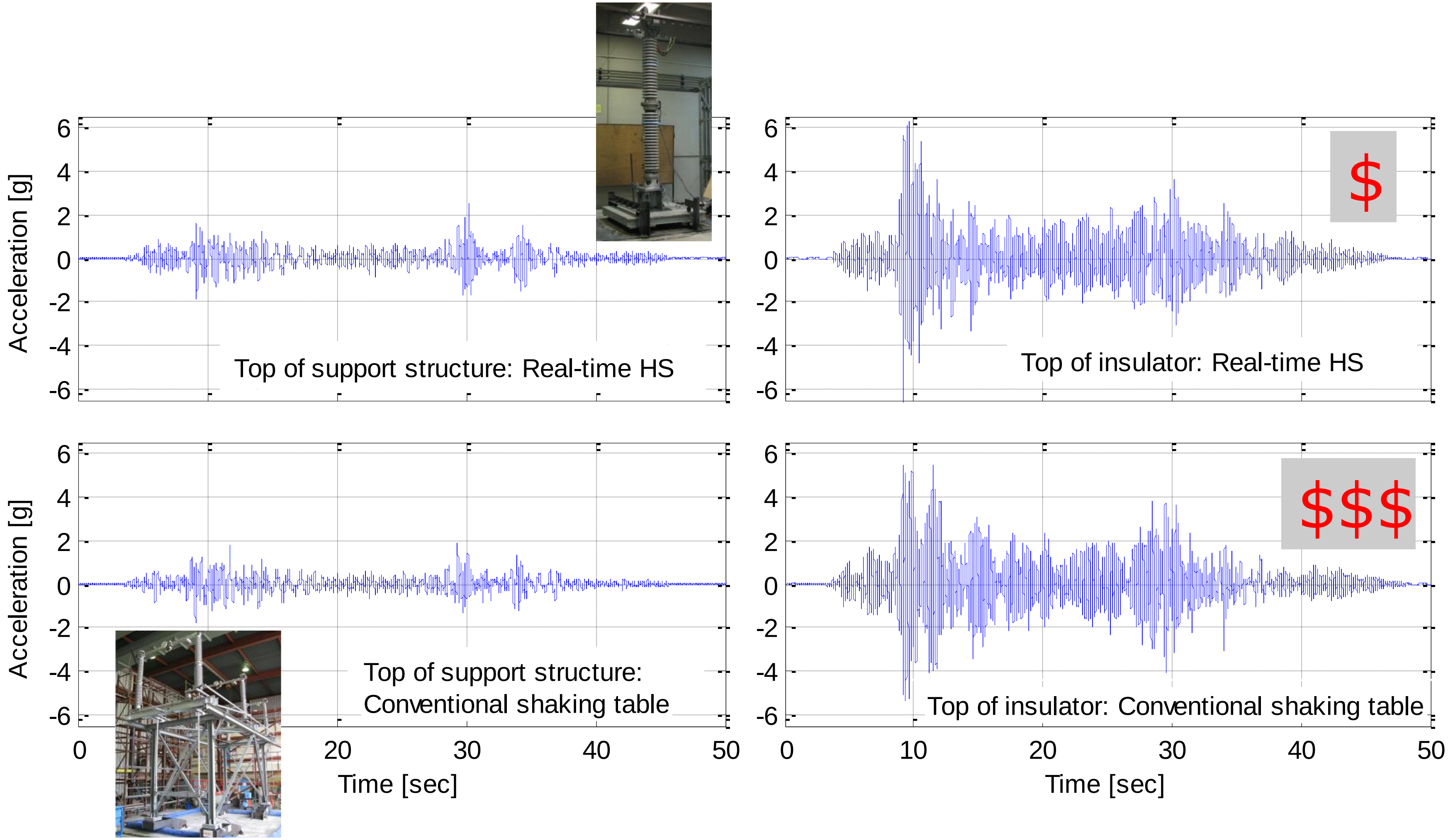
	Case 1	Case 2
Column Reinforcement Ratio (%)	1.5	3.0
Beam Reinforcement Ratio (%)	1.0	3.0
Width of Exterior Columns (mm)	304.8	508
Width of Interior Columns (mm)	355.6	558.8
Depth of Columns (mm)	304.8	457.2
Depth of Beams (mm)	406.4	558.8
Width of Beams (mm)	304.8	406.4

- ❑ Representation of equivalently optimal solutions using Pareto-optimality is very useful for decision makers
- ❑ It provides flexibility to choose among a set of equivalently optimal solutions depending on project requirements
- ❑ The extent to which desired structural performance is satisfied by a selected alternative can be easily observed

Traditional earthquake design is *not sufficient* but *necessary*. Future exercise: Check design of cases 1 & 2 with requirements of seismic codes, e.g. strong column-weak beam, shear failure prevention … etc.)

Optimization-based methods

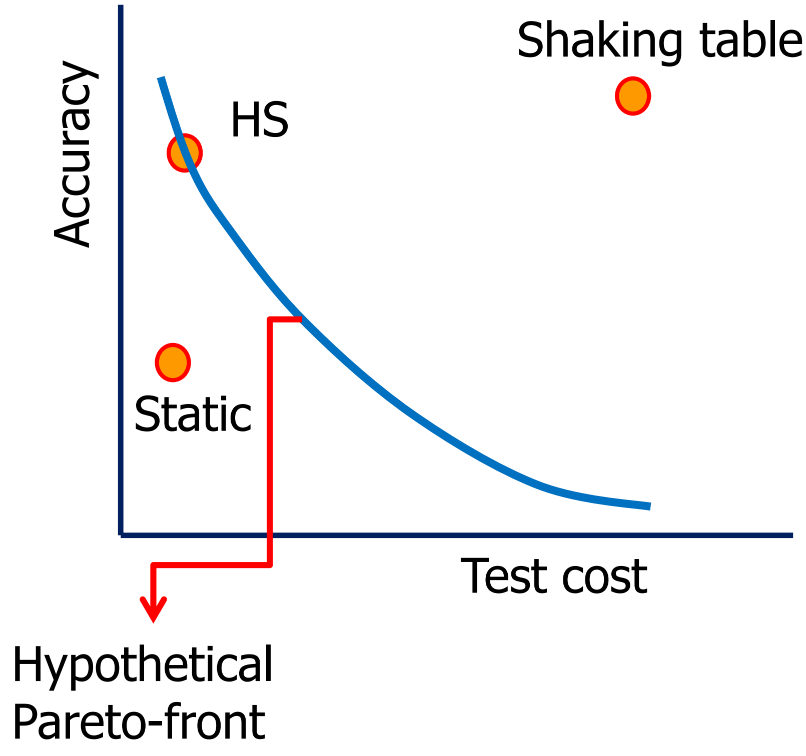
An optimization problem related to hybrid simulation (HS)



Optimization-based methods

An optimization problem related to HS

Qualitative justification of Hybrid Simulation
with an Optimization Technique



Non optimization-based methods

Two available non-optimization-based approaches

- ❑ Krawinkler et al. (2006)
- ❑ Franchin and Pinto (2012)

Krawinkler et al. (2006):

- Can not be considered as a fully probabilistic design procedure
- Iteratively enforces satisfaction of two performance objectives associated with 50/50 and 2/50 hazard levels in terms of cost
- Makes use of median incremental dynamic analysis (IDA) curves [Vamvatsikos & Cornell 2002] to relate the hazard levels with the corresponding EDPs & average loss curves
- The design variables are the fundamental period $\mathbf{T_1}$ & the base shear ratio η (ratio of base shear to weight of the structure).
- Requires a prior production of design-aids in the form of alternative median IDA curves for different values of the design variables.

Non optimization-based methods

Two available non-optimization-based approaches

- ❑ Krawinkler et al. (2006)
- ❑ Franchin and Pinto (2012)

Franchin and Pinto (2012):

- Fully probabilistic
- Employs constraints formulated explicitly in terms of Mean Annual Frequency (MAF) of exceedance of chosen performance-levels
- Can be considered as an approximate method relying on validity of the following:
  - ✓ Closed-form expression for MAF of exceedance of a limit-state [Cornell et al., 2002]
  - ✓ Equal-displacement rule [Veletsos & Newmark, 1960]
- Difference with respect to the optimization approaches: Method produces a solution that is feasible, i.e. that complies with constraints, but not necessarily optimal
- Extension to include an objective function related to, e.g. minimum cost, is possible

Questions?

mosalam@berkeley.edu

http://www.ce.berkeley.edu/people/faculty/mosalam

I-3 PEER PBEE Formulation Demonstrated for Electric Substation Equipment

KHALID M. MOSALAM, PROFESSOR

UNIVERSITY OF CALIFORNIA, BERKELEY

Course Outline 1/2

Part I:

1. PBEE assessment methods
   - ✓ Conditional probability approaches such as SAC/FEMA & PEER formulations
   - ✓ Unconditional probabilistic approach

Questions

2. PBEE design methods
   - ✓ Optimization-based methods
   - ✓ Non optimization-based methods

Questions

3. PEER PBEE formulation demonstrated for electric substation equipment
   - ✓ Introduction
   - ✓ Hazard analysis
   - ✓ Structural analysis
   - ✓ Damage analysis
   - ✓ Loss analysis
   - ✓ Combination of analyses

Questions

Outline

1. Introduction
2. Hazard Analysis
3. Structural Analysis
4. Damage Analysis
5. Loss Analysis
6. Combination of Analyses

Introduction

- **Traditional earthquake design (TED) philosophy:**
  - ➢Prevent damage in low-intensity EQ
  - ➢Limit damage to repairable levels in medium-intensity EQ
  - ➢Prevent collapse in high-intensity EQ

- **TED is necessary but not sufficient as evidenced by:**
  - ➢1994 Northridge and 1995 Kobe earthquakes (initial realizations)

    Unacceptably high damage, economic loss due to downtime & repair cost of structures
  - ➢2009 L'Aquila & 2010 Chile earthquakes (recent evidences)
    - A traditionally designed hospital building evacuated immediately after L'Aquila EQ, while ambulances were arriving with injured people
    - Some hospitals evacuated due to non-structural & infill walls damage after Chile EQ
    - Some residents rejected to live in their homes despite satisfactory performance according to available codes

Introduction

- **First generation PBEE methods:**

Improvement to TED by introducing "Performance Objectives":

Achieve a desired "System Performance" at a given "Seismic Hazard"

System Performance

Hazard Levels (Return Period)	Fully Operational	Operational	Life Safety	Near Collapse
Frequent (43 years)	●	o	o	o
Occasional (72 years)	Δ	●	o	o
Rare (475 years)	♦	Δ	●	o
Very rare (949 years)		♦	Δ	●

o: unacceptable performance

●: basic safety objective

Δ: essential hazardous objective

♦ : safety critical objective

Introduction

- **First generation PBEE methods - Shortcomings:**

❖Deterministic evaluation of performance: No consideration of uncertainty

❖Element level evaluation: No consistency in engineering demands vs. component performance criteria relationships & No ties to global system performance

❖Results only meaningful to engineers: Reduced contribution of stakeholders in decision process

Introduction

- **Pacific Earthquake Engineering Research (PEER) Center PBEE:**
  - ❖ Improvement of first generation PBEE by introducing:
    - ✓ Calculation of performance in a rigorous probabilistic manner: Consideration of uncertainty
    - ✓ Performance definition with decision variables which reflect the global system performance
    - ✓ Performance definition with decision variables in terms of the direct interest of various stakeholders
    - X Shortcoming: Mostly used by academia with ***little*** attention from practicing engineers. However, there are several examples of recent increased attention from the SF Bay Area design firms.

FEMA-P58: Seismic Performance Assessment of Buildings;

A potential milestone to incorporate PBEE in standard design practice.

Introduction

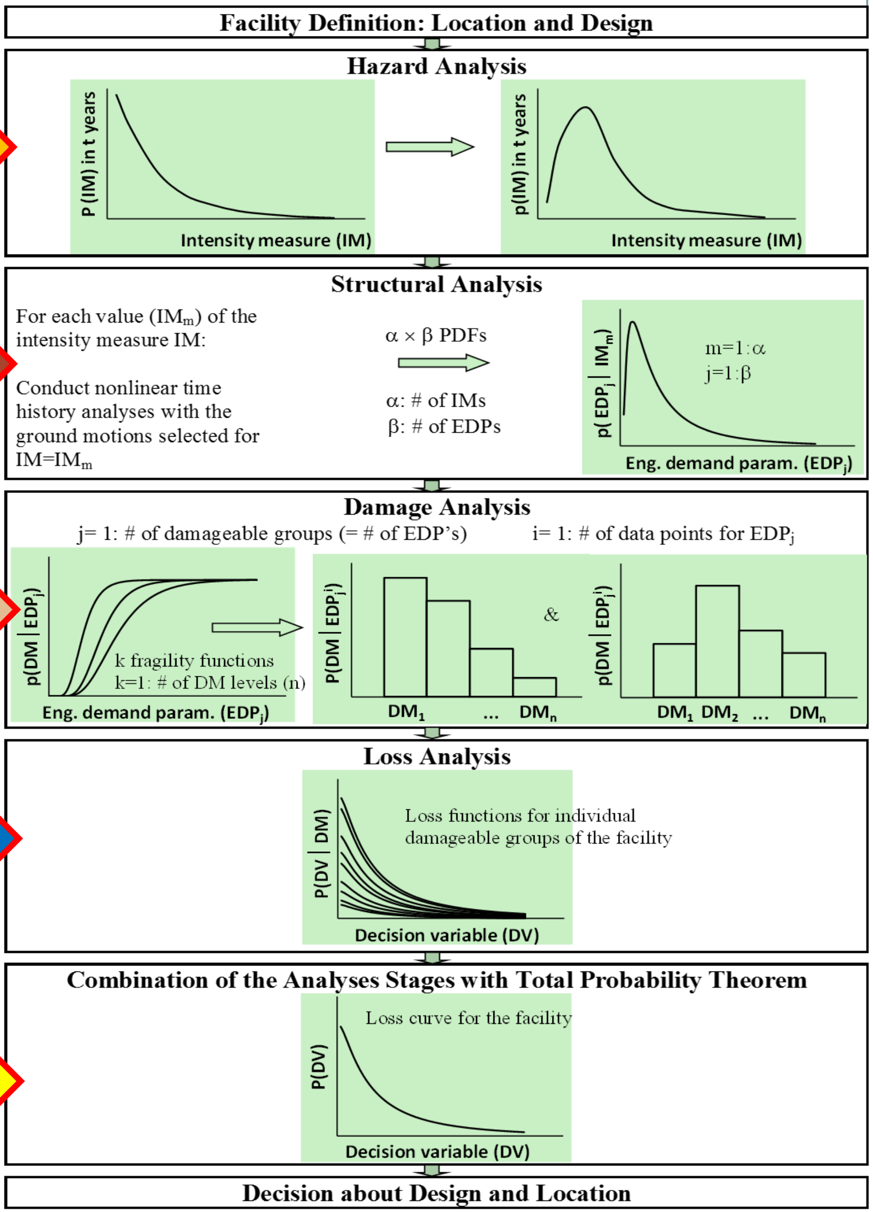
- **PEER PBEE (Revisited):**

  - Gaining popularity of probabilistic Performance-Based Engineering Design (PBED) methods
  - PBED methods likely to be used for standard design codes in the near future
  - Necessity to find paths for popularization of the method within the practicing structural engineering community
  - **Objective**: Explain PEER PBEE methodology in a simplified manner to reach the broader engineering community around the world

PEER PBEE Formulation

- **Hazard Analysis**: Earthquake hazard during the lifecycle of a building (uncertainty in fault locations, magnitude-recurrence rates, level of attenuation, etc.)
- **Structural Analysis**: Response of the structure to the earthquake hazard (uncertainty in ground motion type, material properties, damping, etc.)
- **Damage Analysis**: Level of damage corresponding to the response of the structure (uncertainty in material limit state characteristics, damage pattern & history, etc.)
- **Loss Analysis**: Value of a decision variable (DV, e.g. economic loss) corresponding to damage (uncertainty in damage distribution, variation of components resulting in same damage level, etc.)

End Product: Due to the different sources of uncertainty, there is no single deterministic value of DV. Instead, there are multiple values of DV with varying probability.



PEER PBEE Formulation

Single Damageable Group (e.g. structural) and no global collapse:

POE of the n^th^ value of the DV

$$P\left(DV^n\right)=\sum_m\sum_i\sum_k \underbrace{P\left(DV^n \middle| DM_k\right)}_{\text{Loss}}\,\underbrace{p\left(DM_k \middle| EDP^i\right)}_{\text{Damage}}\,\underbrace{p\left(EDP^i \middle| IM_m\right)}_{\text{Structural}}\,\underbrace{p\left(IM_m\right)}_{\text{Hazard}}$$

Multiple Damageable Groups (e.g. structural & non-structural) and no global collapse:

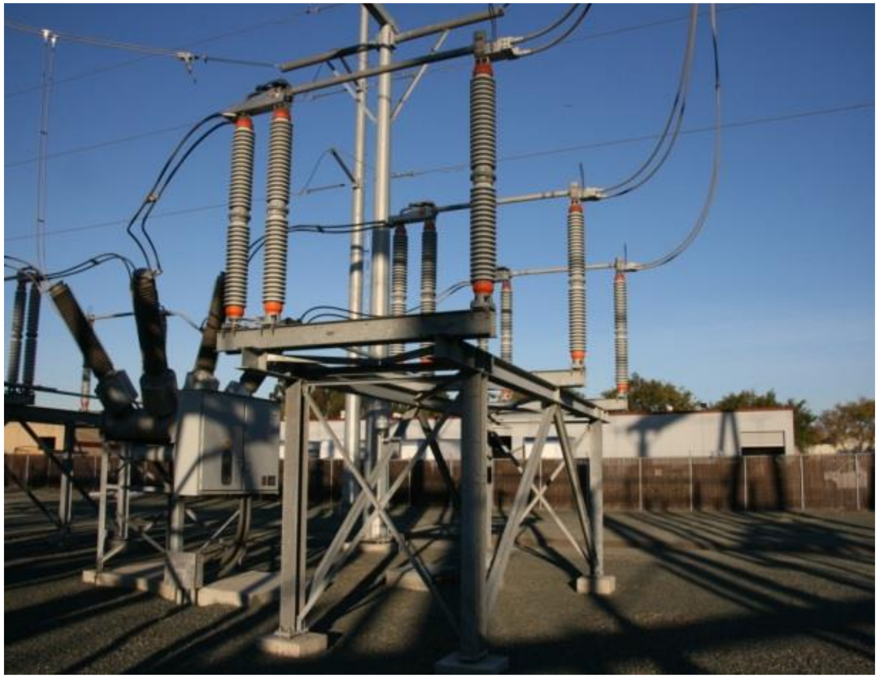
$$P\left(DV^n\right)=\sum_m\sum_j\sum_i\sum_k P\left(DV_j^n \middle| DM_k\right)\,p\left(DM_k \middle| EDP_j^i\right)\,p\left(EDP_j^i \middle| IM_m\right)\,p\left(IM_m\right)$$

Multiple Damageable Groups and global collapse:

$$P\left(DV^n\right)=\sum_m\left(\sum_j\sum_i\sum_k P\left(DV_j^n \middle| DM_k\right)\,p\left(DM_k \middle| EDP_j^i\right)\,p\left(EDP_j^i \middle| IM_m\right)p\left(NC \middle| IM_m\right)+P\left(DV^n \middle| C\right)p\left(C \middle| IM_m\right)\right)\,p\left(IM_m\right)$$

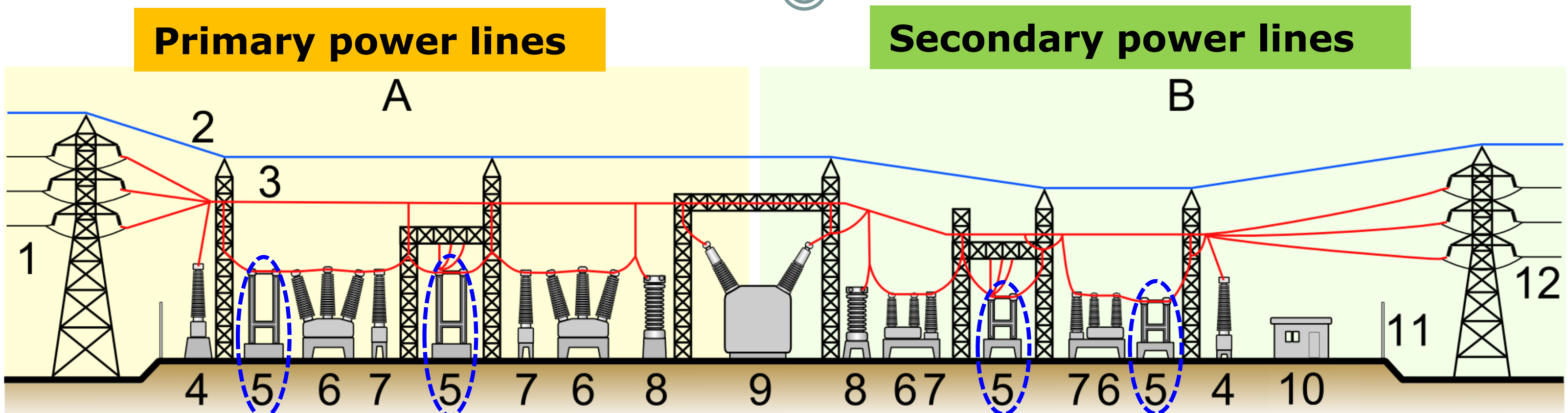
Commonly utilized DVs: Fatalities, Economic loss & Repair duration

Demonstrative Application



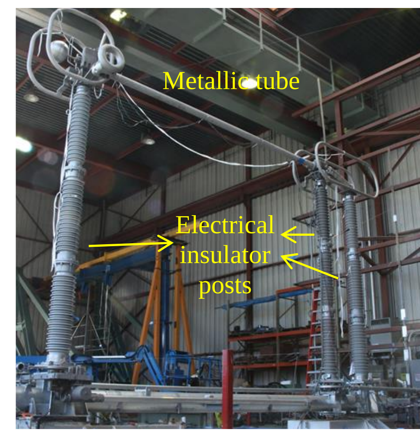
230 kV Disconnect Switches in Substations

Application



* Courtesy of Wikipedia

1. Primary power lines
2. Ground wire
3. Overhead lines
4. Transformer
5. Disconnect switch
6. Circuit breaker
7. Current transformer
8. Lightning arrester
9. Main transformer
10. Control building
11. Security fence
12. Secondary power lines



Disconnect switches are key components of power transmission & distribution systems.

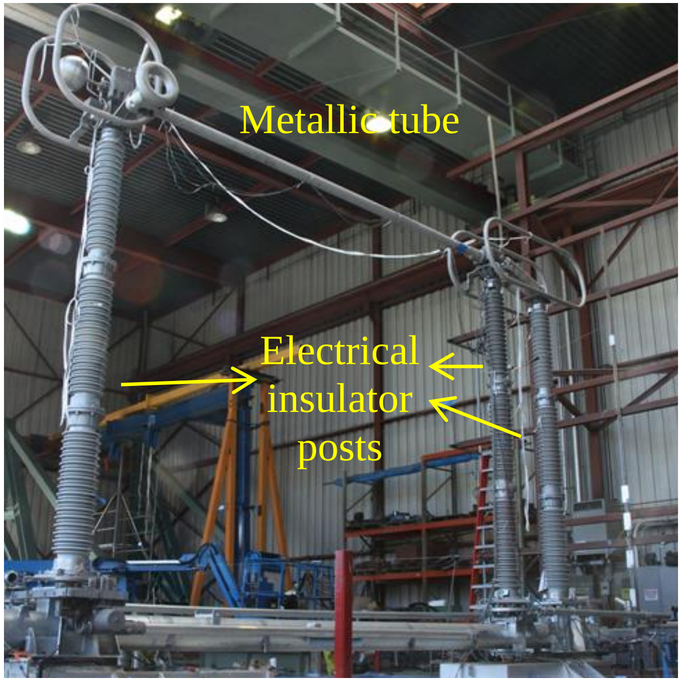
Major elements of an electrical substation (distribution substation shown)

Application

- Disconnect switches are used to control flow of electricity between substation equipment, e.g. to interrupt power during maintenance.

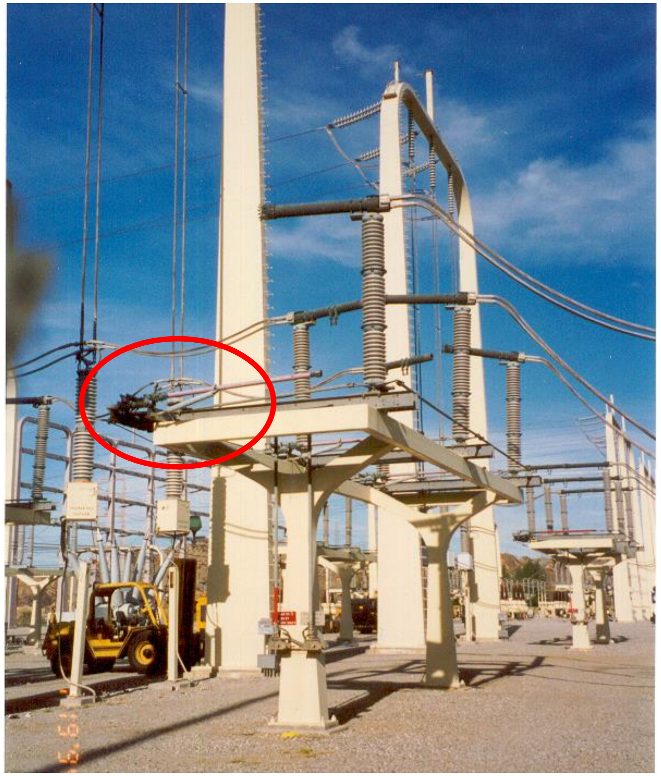
- They are used to manage power distribution network, e.g. shifting loads across network or turning off part of the network for safety.

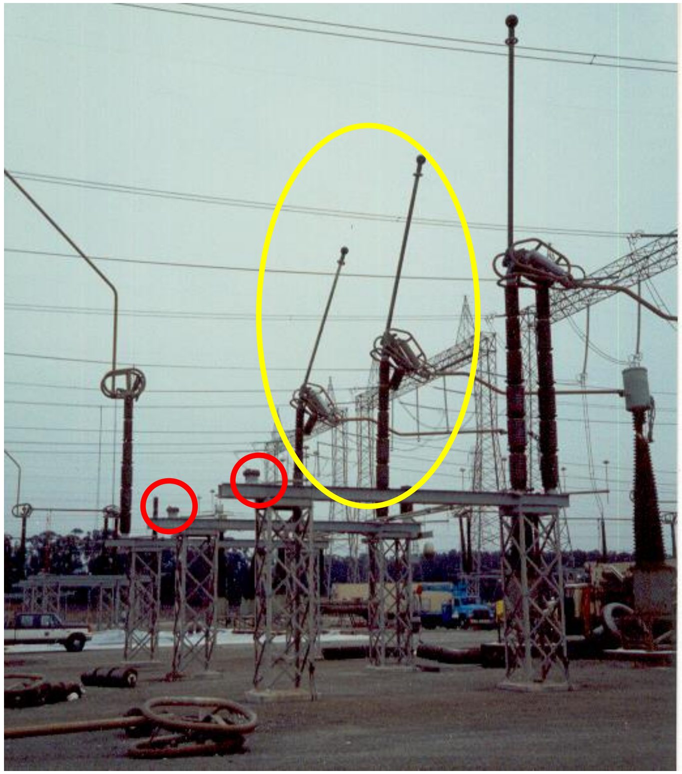
- Proper functioning of the disconnect switches is vital for power regulation in the aftermath of an earthquake.



Application

Disconnect Switch and Insulator Damage during Earthquakes





Courtesy of Eric Fujisaki, PG&E

Application

Ertaishan Switchyard (220kV) Destruction (PGA ~ 0.5g), Yingxiu Town
Wenchuan Earthquake, China, May 12, 2008 **[Photo credit: Q. Xie, Tongji University]**

Hazard Analysis

- First analysis stage in PEER PBEE formulation
- A natural hazard is a threat of a naturally occurring event that will have a negative effect on people or the environment:
  - Earthquakes
  - Volcanoes
  - Hurricanes
  - Landslides
  - Floods or droughts
  - Wildfires
- PEER PBEE considers earthquake hazard (seismic hazard)

Hazard Analysis

- <u>Uncertainty in seismic hazard</u>:

  a. Potential fault locations

  b. Magnitude-recurrence rates

  c. Level of attenuation

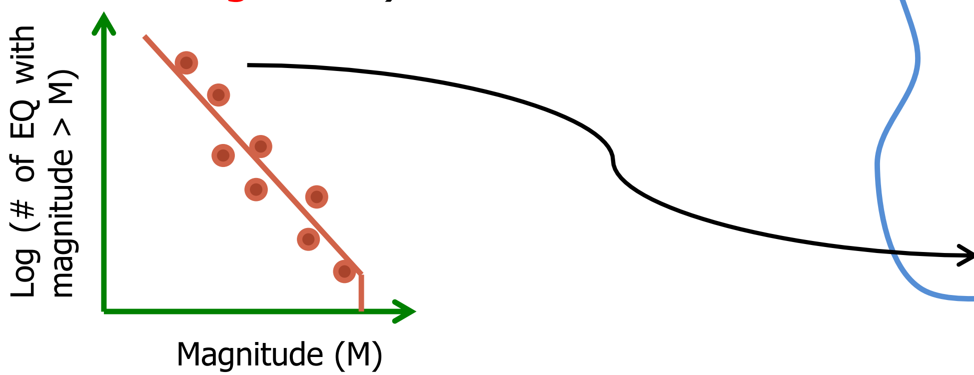
- Deterministic Seismic Hazard Analysis (Limited uncertainty consideration: only item "**c**" above)

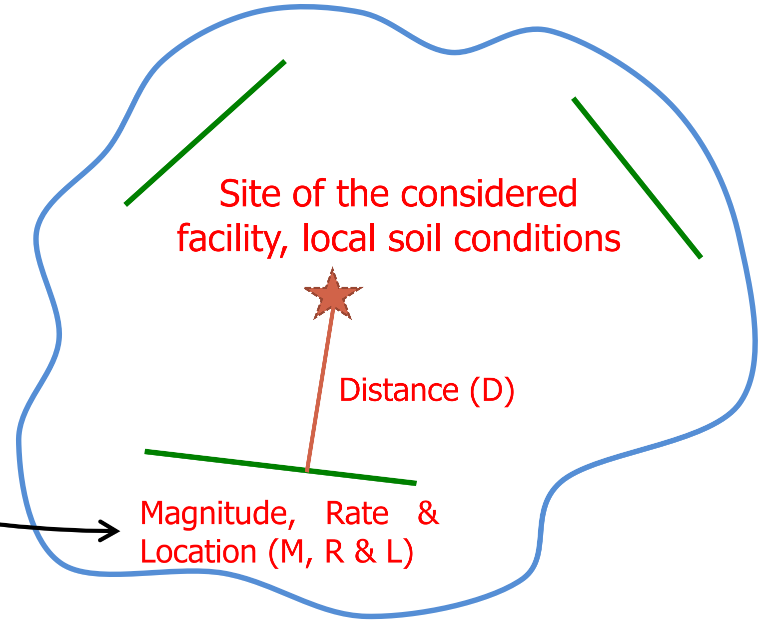
- Probabilistic Seismic Hazard Analysis (Complete uncertainty consideration → <u>Preferred method</u>)

Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

1. Determine the potential fault locations
2. Determine the magnitude-recurrence relationships for the faults (rate of each possible magnitude)





3. For all the potential earthquake scenarios (M, R & L):
   - Using ground motion prediction equations: Calculate the mean and standard deviation (μ & σ) of an intensity measure (IM) as a function of (M, D)
   - Determine the probability distribution function (PDF) and probability of exceedance (POE) of IM using μ & σ
   - Multiply POE with R to determine annual frequency of exceedance (AFE) of IM

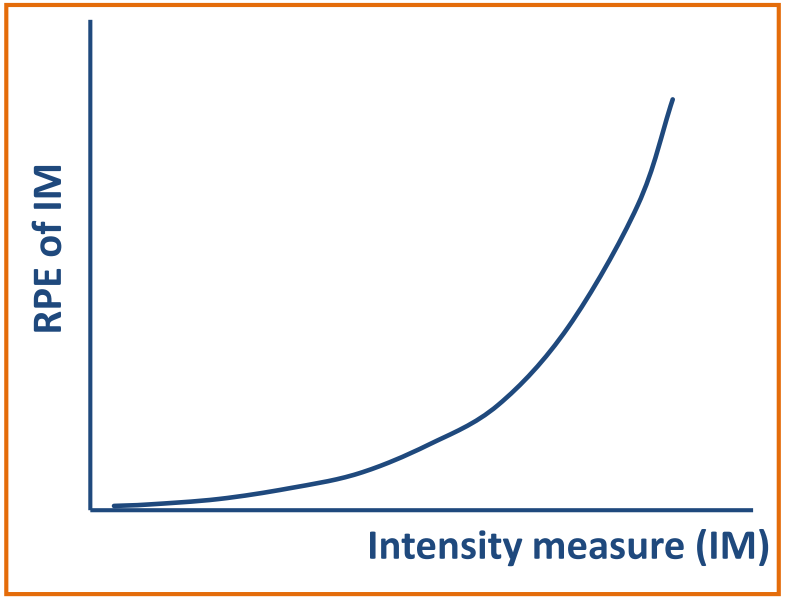
Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

4. Sum AFE from all scenarios to obtain the total annual frequency of exceedance (TAFE) of IM

An easier way of representing TAFE: Return period of exceedance,

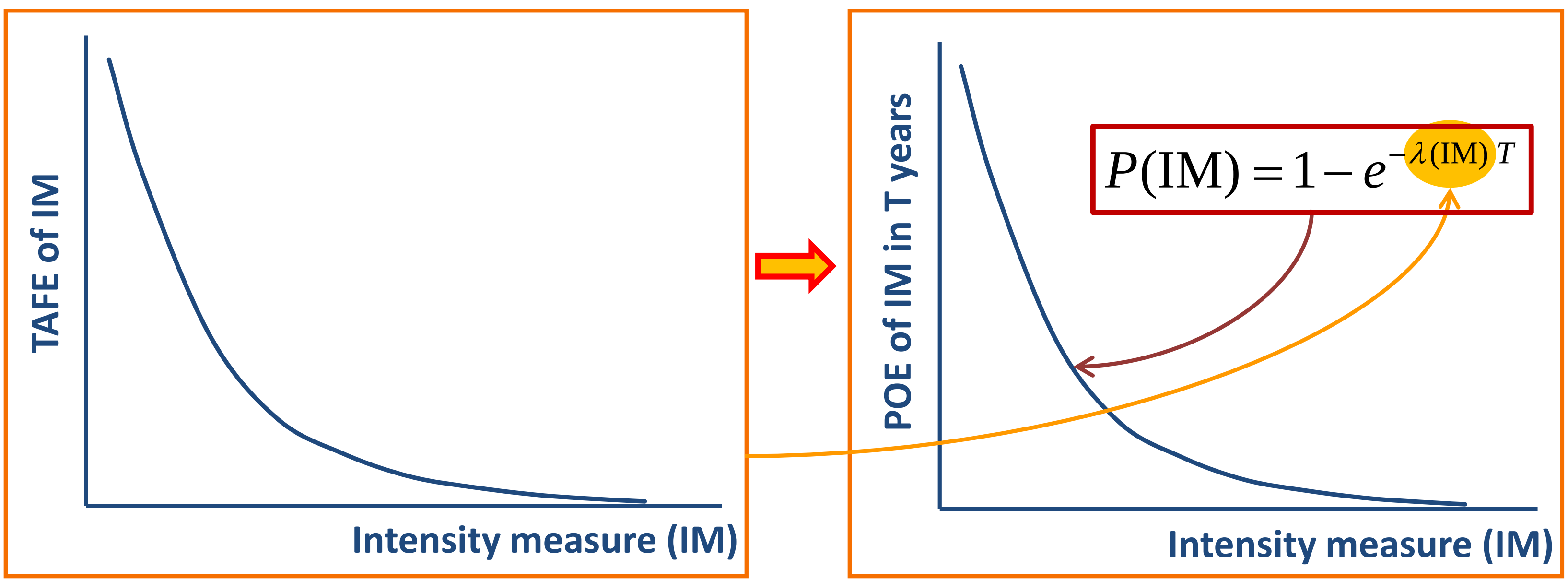
RPE = 1/TAFE



Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

5. From Poisson's model, calculate POE of IM in T years from TAFE

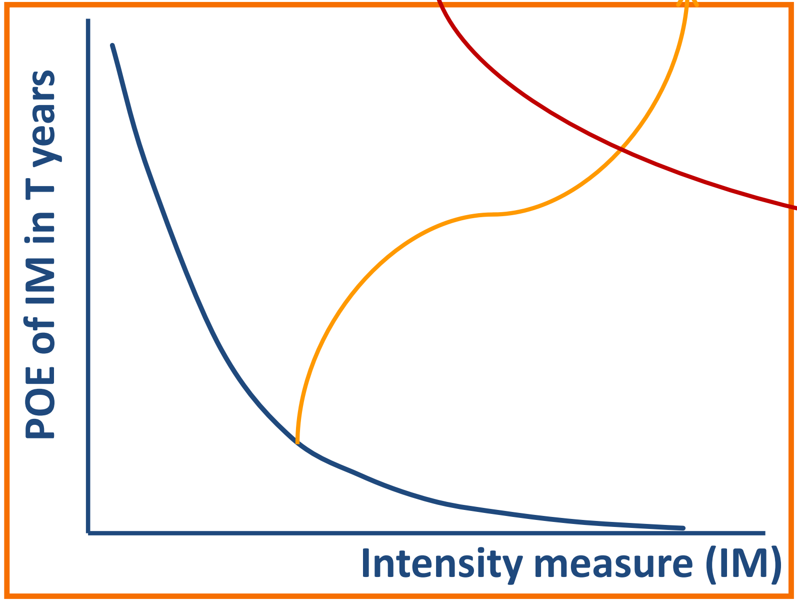


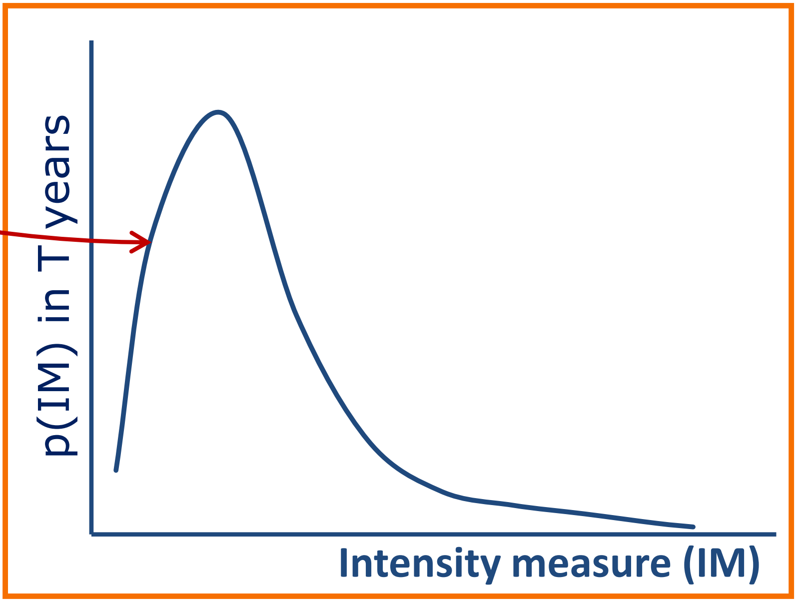
Hazard Analysis

Probabilistic Seismic Hazard Analysis (PSHA)

6. Calculate probability of IM in T years from POE

$$
\begin{aligned}
&\text{for } m = 1:\#\text{ of IM data points}\\
&p(IM_m) = P(IM_m) \quad \text{if } m = \#\text{ of IM data points}\\
&p(IM_m) = P(IM_m) - P(IM_{m+1}) \quad \text{otherwise}
\end{aligned}
$$

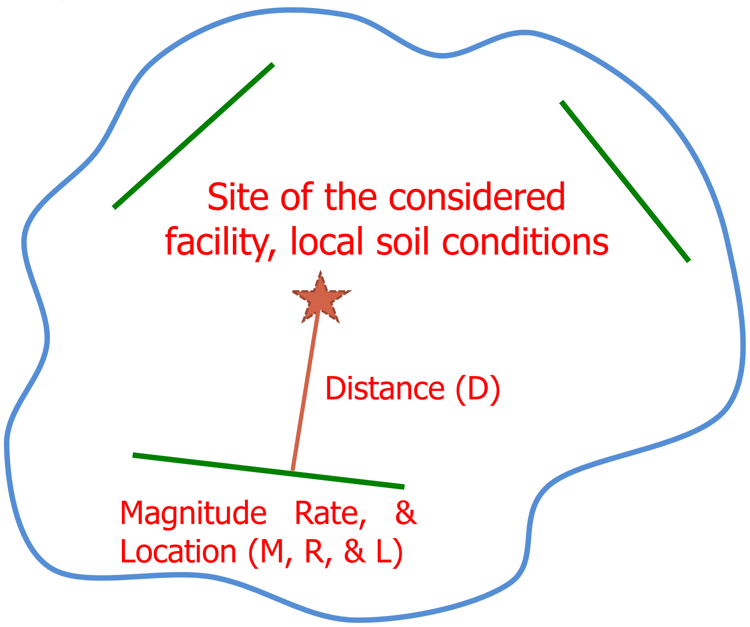




Hazard Analysis

Deterministic Seismic Hazard Analysis (DSHA)

1. & 2. as PSHA
3. For one or only few (generally the most critical) of the potential earthquake scenarios (M, R, & L)
   - Determine the value of intensity measure (IM) as a function of (M, D)
   - Inherent consideration of uncertainty due to the probabilistic nature of ground motion prediction equations



Hazard Analysis

- **Outcome of hazard analysis**: Probability of exceedance (POE) and probability (p) of Intensity Measure (IM)
- **Commonly used IMs**:
  - Peak ground acceleration [PGA]
  - Peak ground velocity [PGV]
  - Spectral acceleration at fundamental period [$S_a(T_1)$]

Reason of common use: Ground motion predictions available

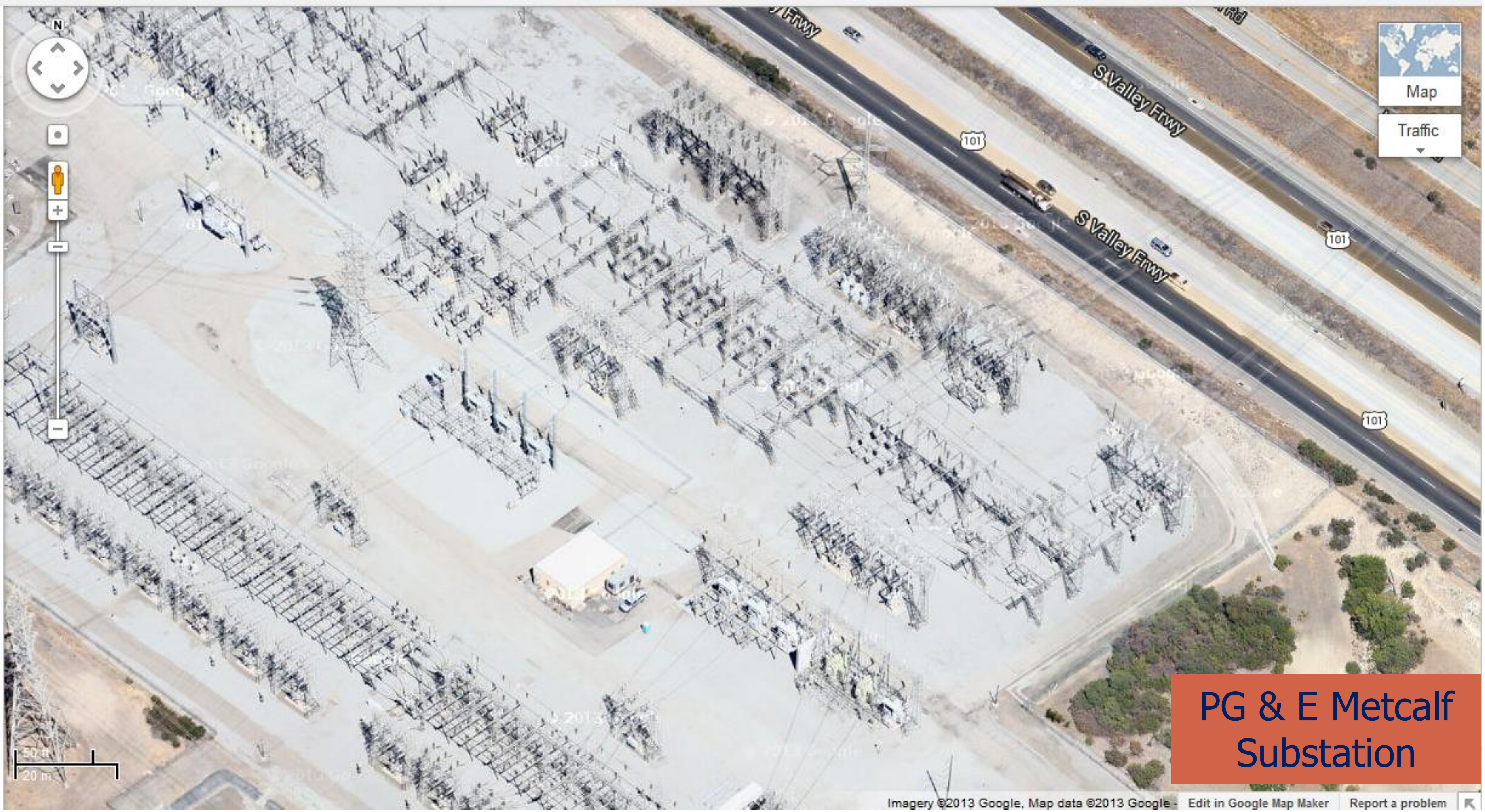
- **Alternatives for IM** [e.g., Tothong and Cornell (2007)]:
  - Inelastic spectral displacement
  - Inelastic spectral displacement with a higher-mode factor

Hazard Analysis

- **Selection of ground motion (GM) time histories**: Compatible with the hazard curve for each intensity level (i.e. each IM value)

  - Adequate number of GMs to provide meaningful statistical data in the structural analysis phase
  - GMs compatible with the magnitude and distance pair which dominates the hazard
  - Use of unscaled GMs whenever possible
  - Separation of unscaled ground motions into bins: Performed once and used for consecutive cases

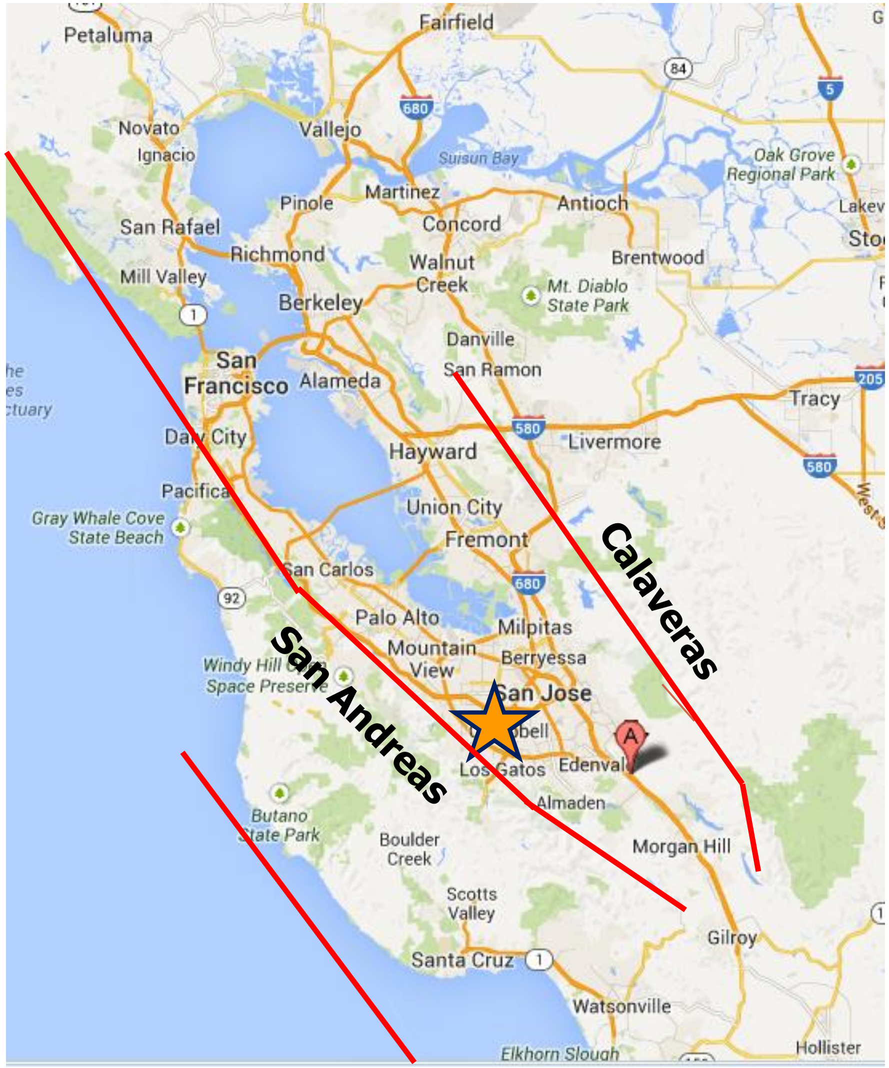
Hazard Analysis: Application



PG & E Metcalf Substation

Hazard Analysis: Application

PG&E Metcalf Substation:



Location of the structure:
San Jose, California
(37.226°, -121.744°)

Site class: NEHRP D

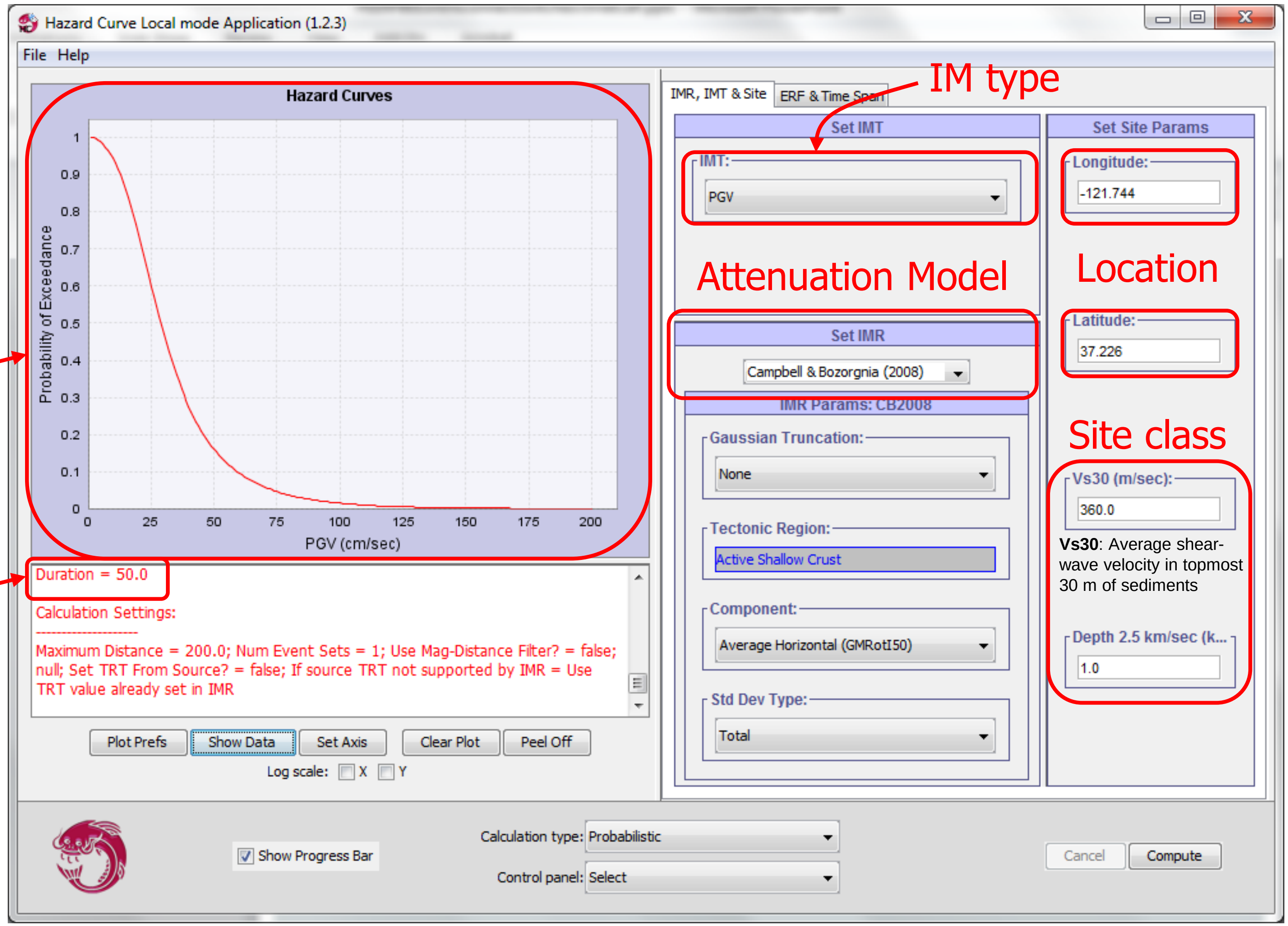
Hazard Analysis: Application

Hazard Analysis

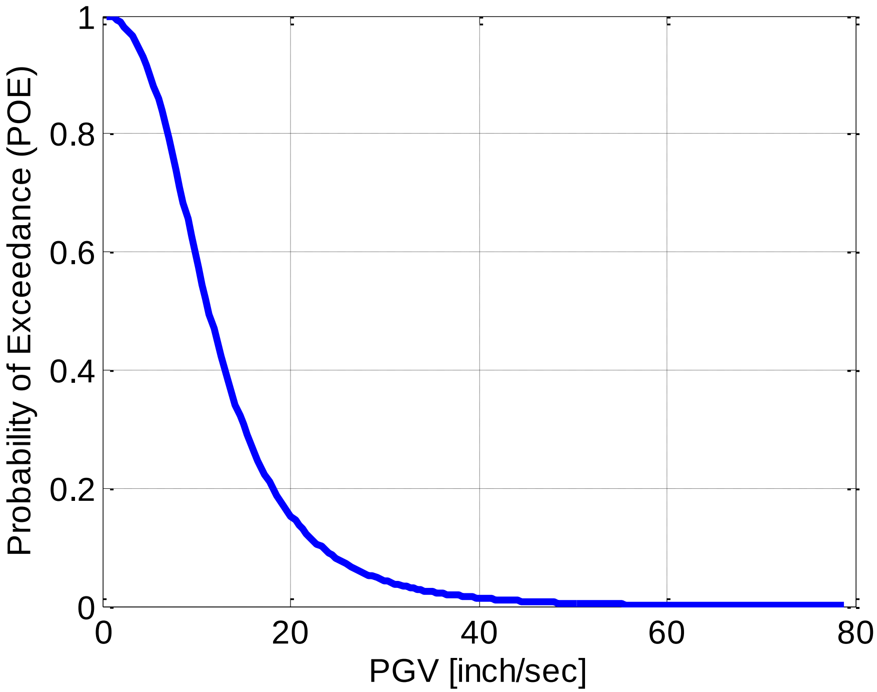
OpenSHA
http://www.opensha.org

Hazard Curve

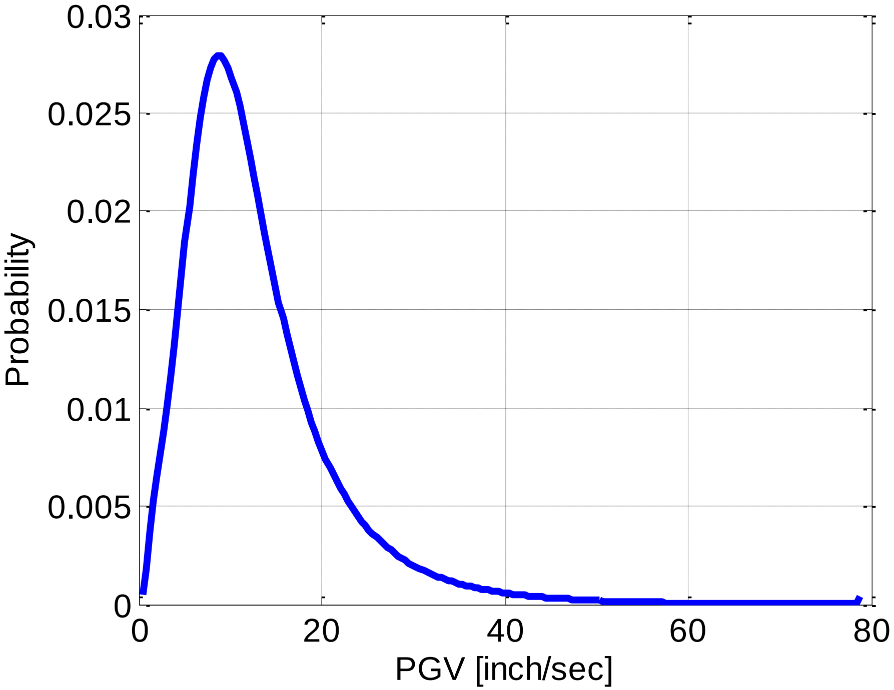
Duration (years)



Hazard Analysis: Application



Hazard Curve:
POE of PGV



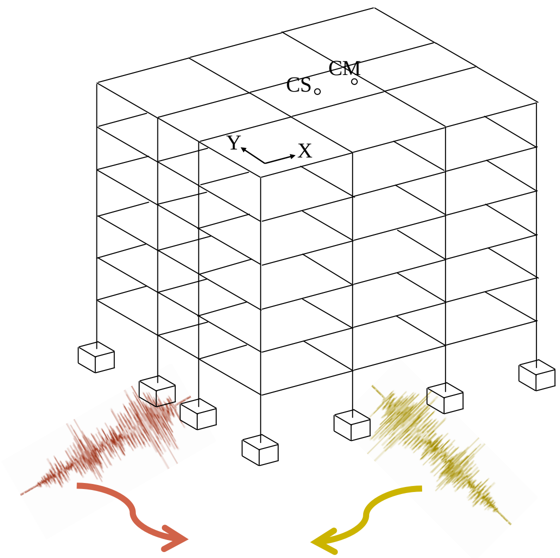
Hazard Curve:
Probability of PGV

$$P\left(DV^{n}\right)=\sum_{m}\sum_{j}\sum_{i}\sum_{k}P\left(DV_{j}^{n}\middle|DM_{k}\right)p\left(DM_{k}\middle|EDP_{j}^{i}\right)p\left(EDP_{j}^{i}\middle|IM_{m}\right)p\left(IM_{m}\right)$$

Structural Analysis

❖ Second analysis stage in PEER PBEE Formulation

❖ A computational model of the structure:



GMs from hazard analysis
(*uncertainty in GM characteristics*)

Uncertainty in

- Mass (e.g. variation in live load)
- Damping (e.g. epistemic uncertainty in damping models)
- Material characteristics (e.g. strength, ultimate strain)

❖ Nonlinear time history simulations with ground motions from hazard analysis

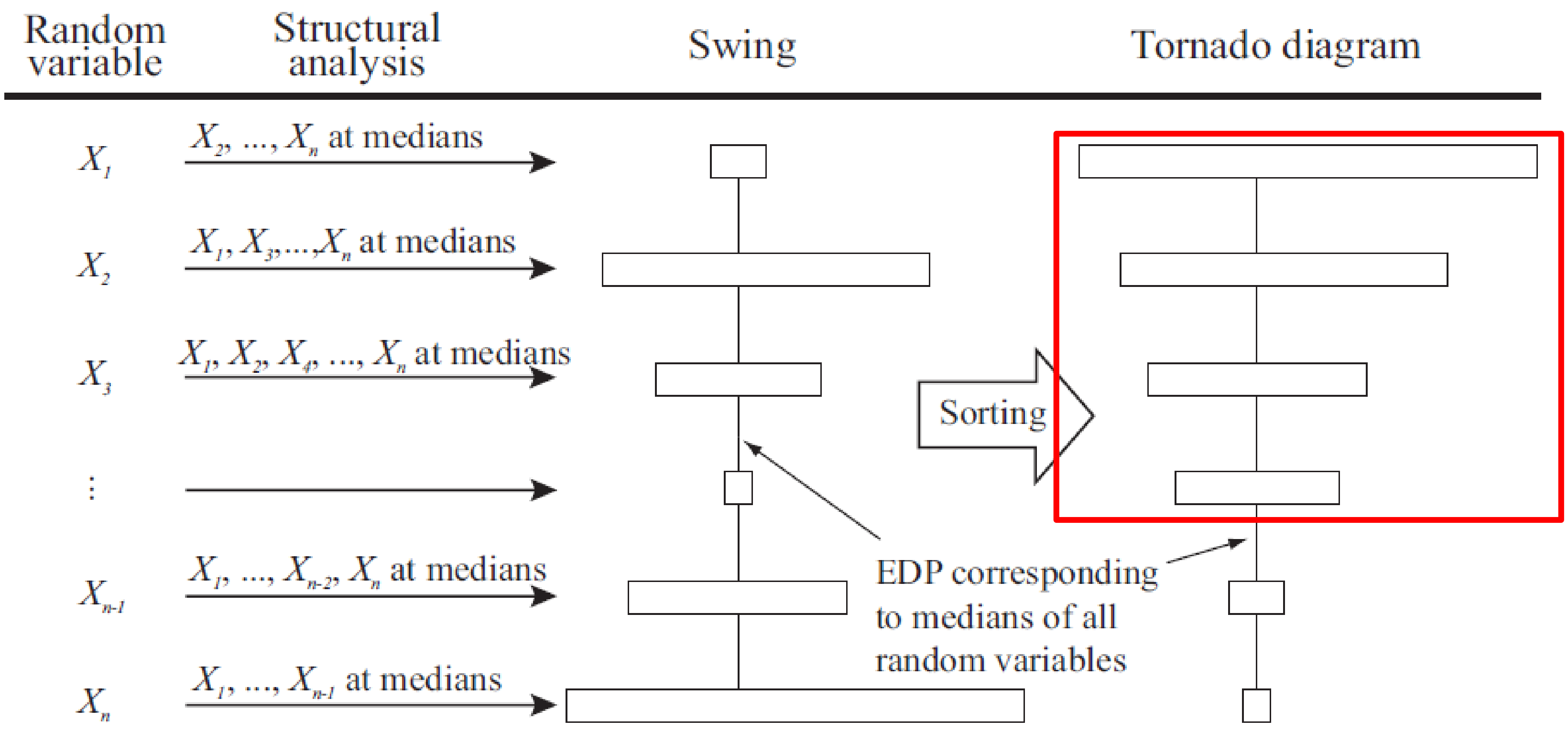
Structural Analysis

❖ **Potential variables in analyses**:

- Ground motion
- Mass
- Damping ratio
- Damping model
- Strength
- Modulus of elasticity
- Ultimate strain

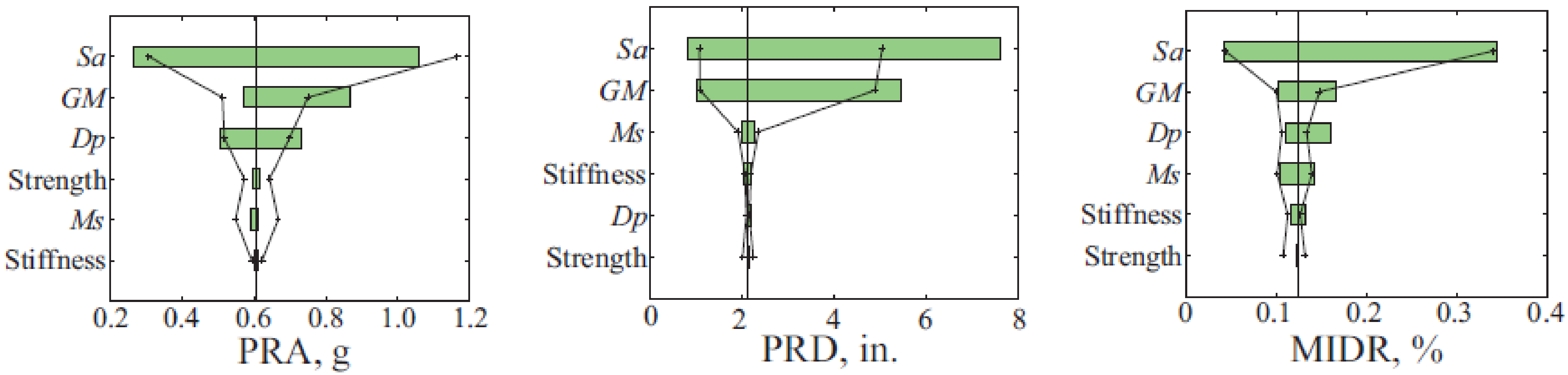
Structural Analysis

❖ **Determination of** **important** **variables**: Tornado diagram analysis (Lee and Mosalam, 2006)



Structural Analysis

- **Determination of** **important** **variables**: Tornado diagram analysis (Lee and Mosalam, 2006)

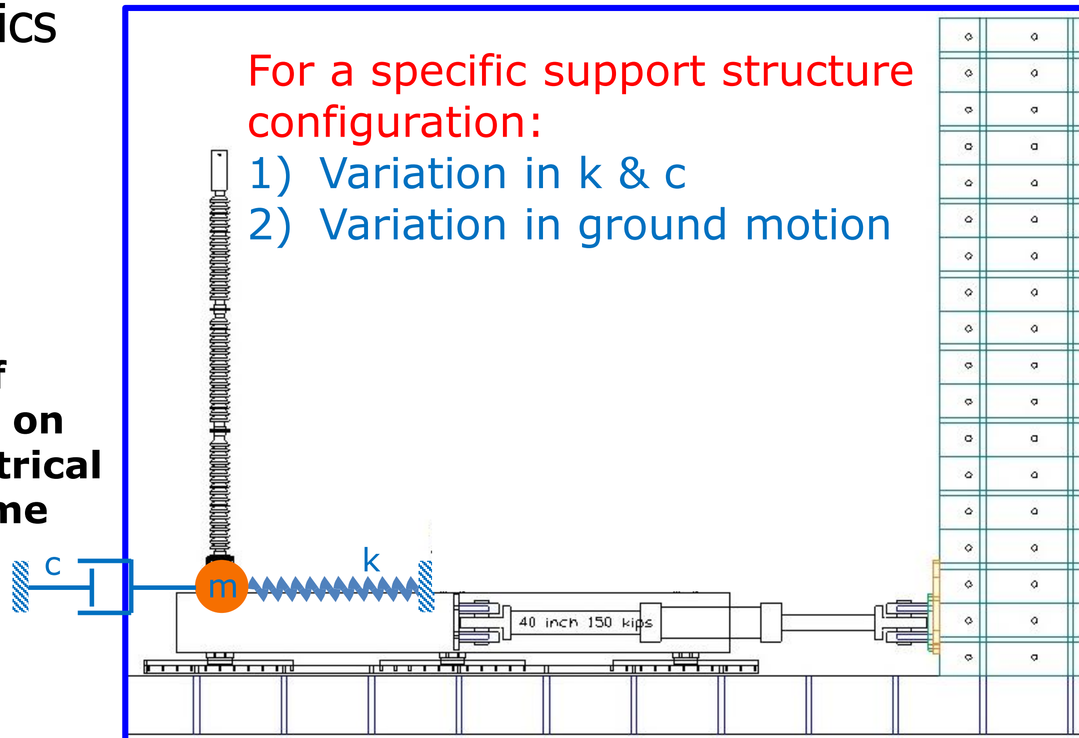


- Determine the variables with negligible effect on the structural response variability and reduce the number of simulations by eliminating unnecessary sources of uncertainties

Structural Analysis

- Recall Hybrid Simulation (from past workshop)
  - In some cases, hybrid simulation can be an alternative to the nonlinear time history simulations
  - For example, elimination of the simulations for the uncertainties in material characteristics

Investigation of the Effect of support structure properties on the seismic response of electrical insulator posts using real-time hybrid simulation (RTHS)



Structural Analysis

- **Structural analysis outcome**: Engineering Demand Parameter (EDP)
- Local parameters: e.g. element forces & deformations
- Global parameters: e.g. floor acceleration & interstory drift
- **Different EDPs for different damageable groups**:
  - Axial or shear force in a non-ductile column
  - Plastic rotations for ductile flexural behavior

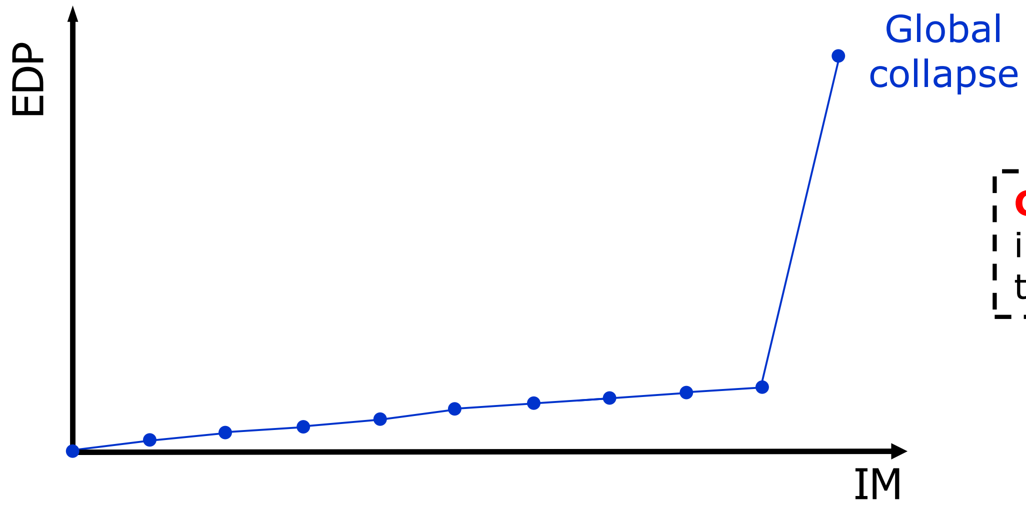
  } structural components
  - Floor acceleration: non-structural components
  - Interstory drift: structural & non-structural components
- **Peak values** of the above EDPs

Structural Analysis

❖ Separate treatment of global collapse since its probability does not change from one damageable group to another.

Methods of global collapse determination

Method I: Scaling a set of GMs for each intensity level



Global collapse: Unrealistic increase of EDP corresponding to a small increase in IM

Probability of global collapse for an intensity level:

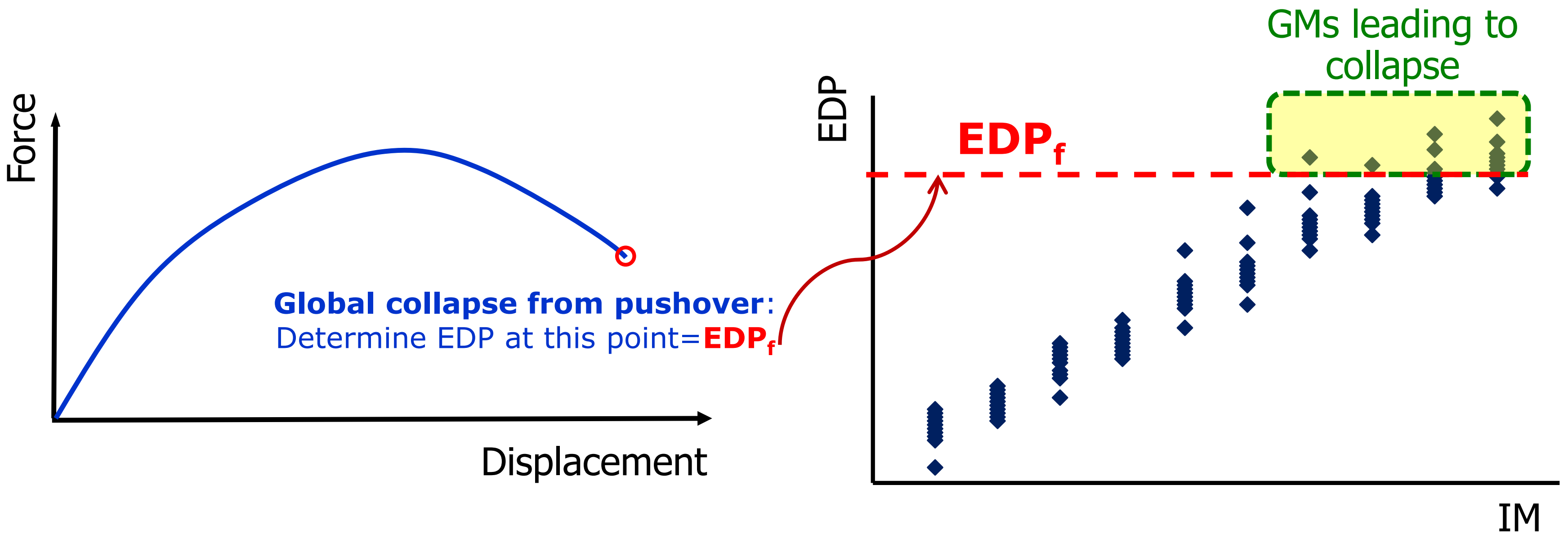
p(C|IM) = # of GMs leading to collapse / total # of GMs

Structural Analysis

❖ Separate treatment of global collapse since its probability does not change from a damageable component to another.

Methods of global collapse determination

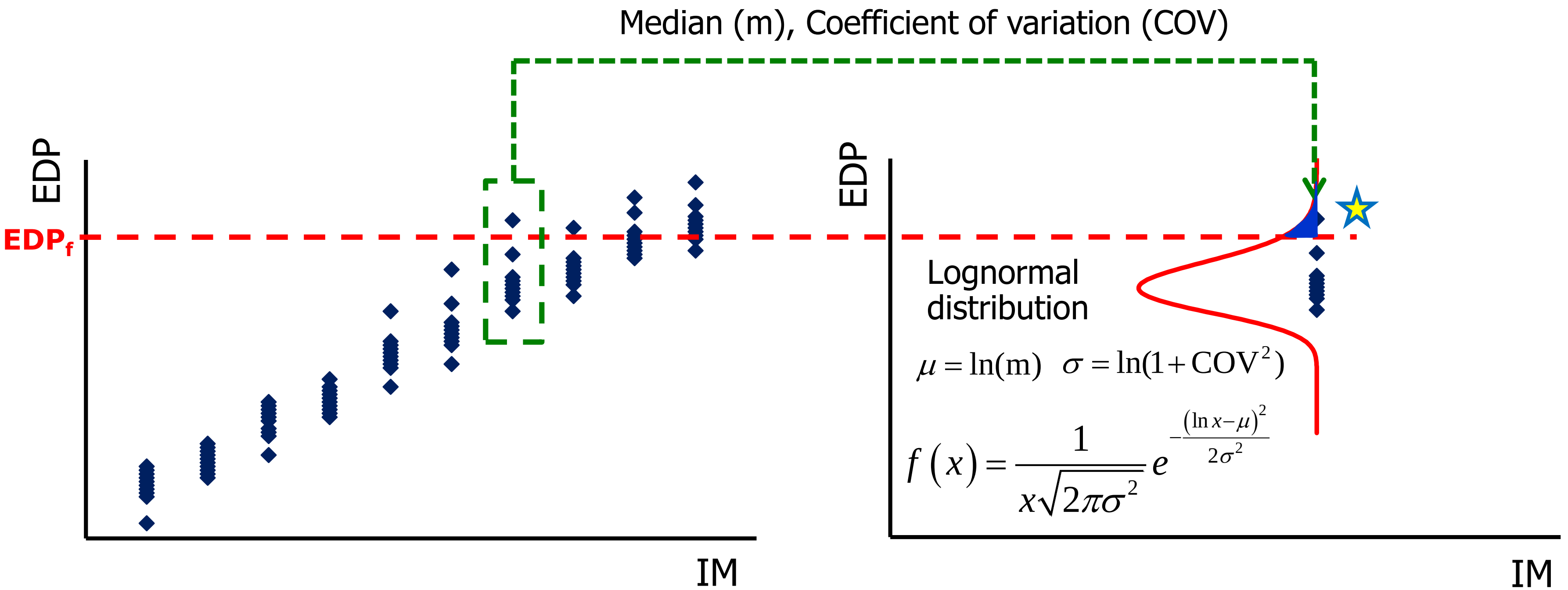
Method II: Use of unscaled GMs



Structural Analysis

Methods of global collapse determination

Method II: Use of unscaled GMs

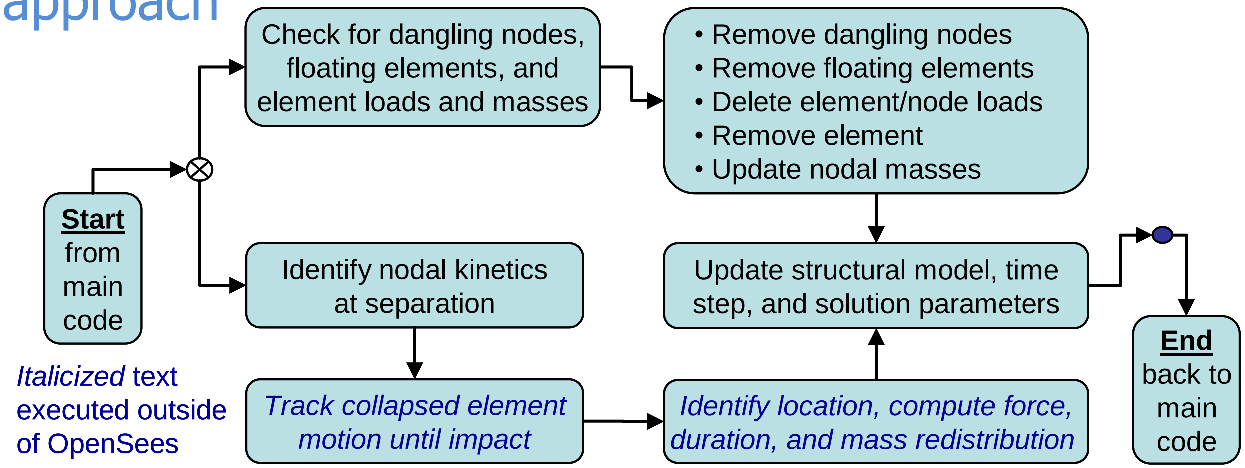


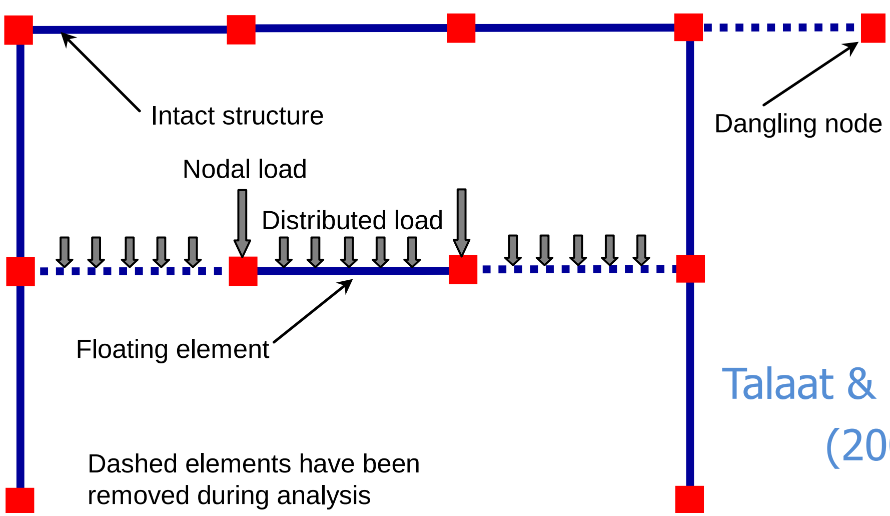
a) p(C|IM) = # of GMs leading to collapse / total # of GMs (preferred method)

b) p(C|IM) = shaded area ☆

Structural Analysis

Progressive Collapse: A realistic representation of collapse in OpenSees using element removal approach





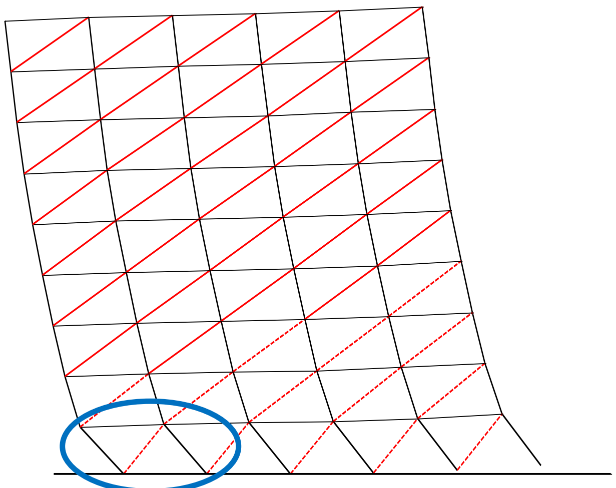
Talaat & Mosalam (2008)

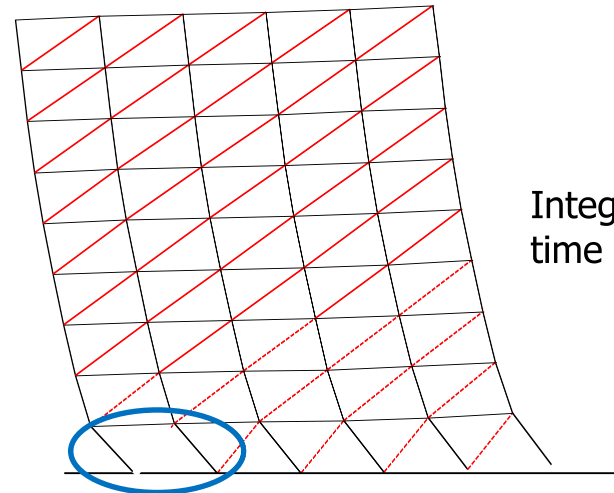
OpenSees: Open System for Earthquake Engineering Simulation
http://opensees.berkeley.edu/

Structural Analysis

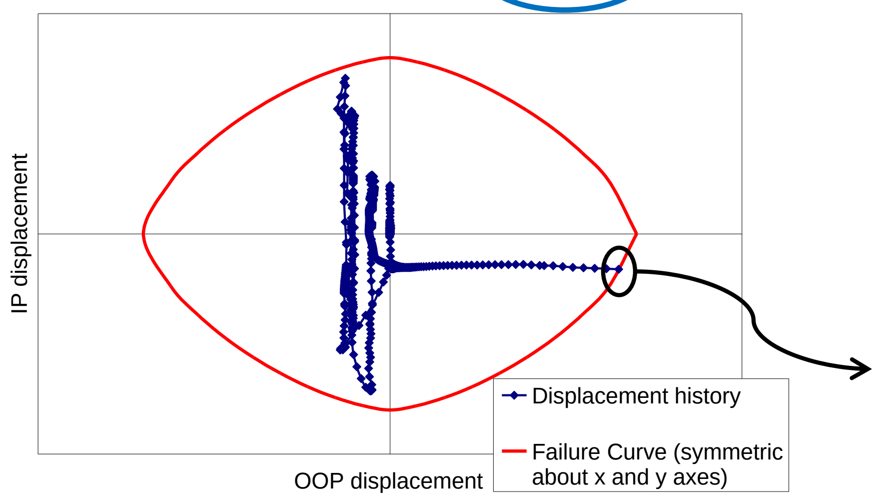
Progressive Collapse: A realistic representation of collapse in OpenSees

Integration
time step i-1





Integration
time step i

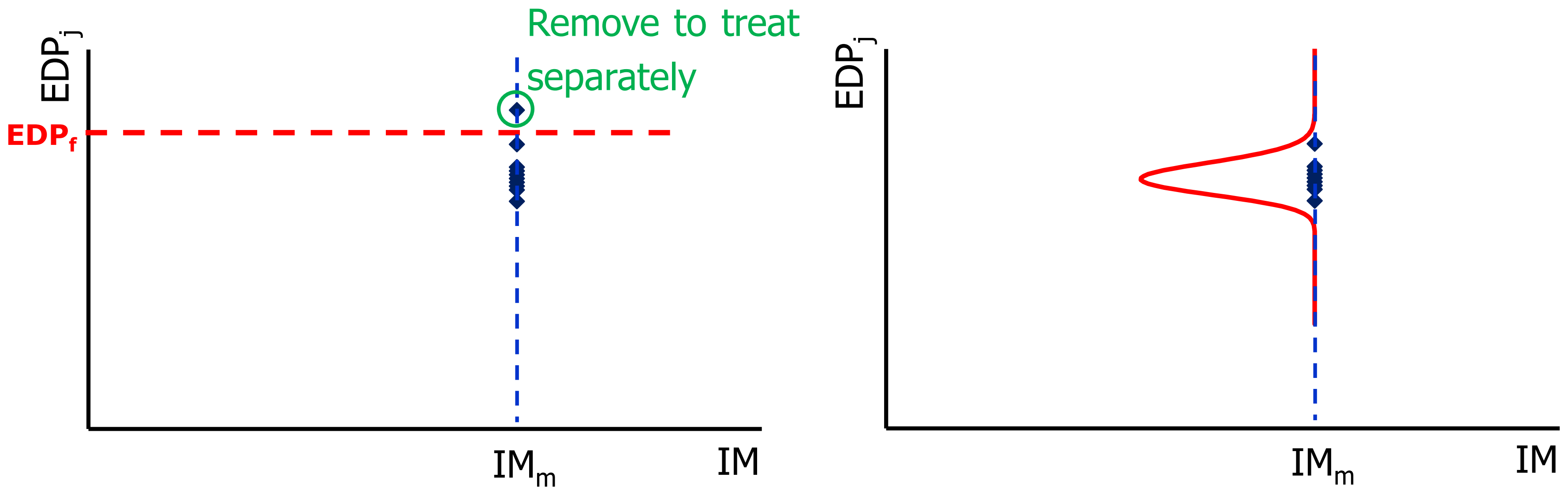


@Integration
time step i

http://opensees.berkeley.edu/wiki/index.php/Infill_Wall_Model_and_Element_Removal

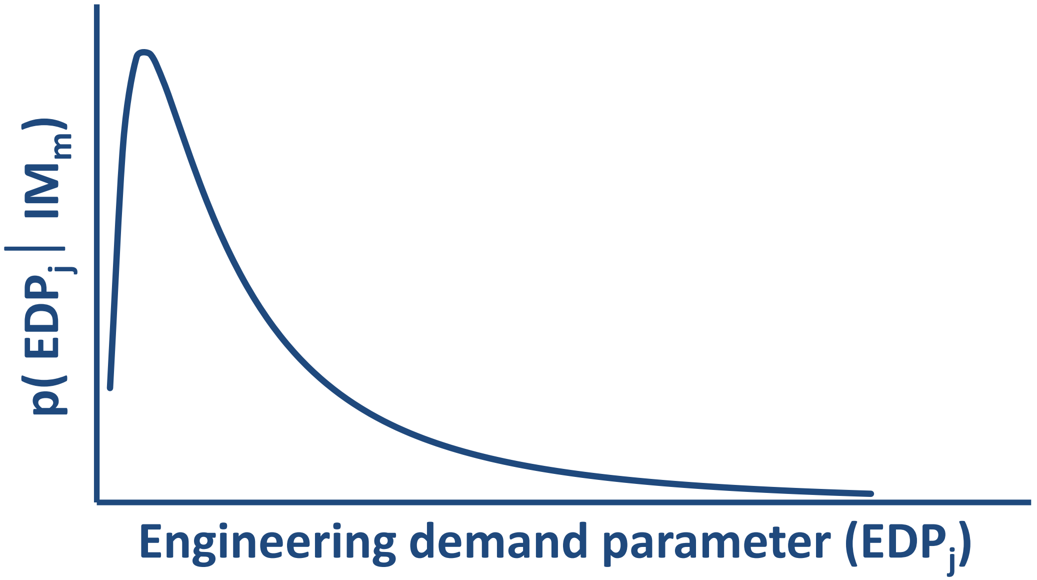
Structural Analysis

Outcome of Structural Analysis: Probability of each value (index i) of each EDP (index j) for each hazard level (index m): $\mathbf{p(EDP_j^i \mid Im_m)}$



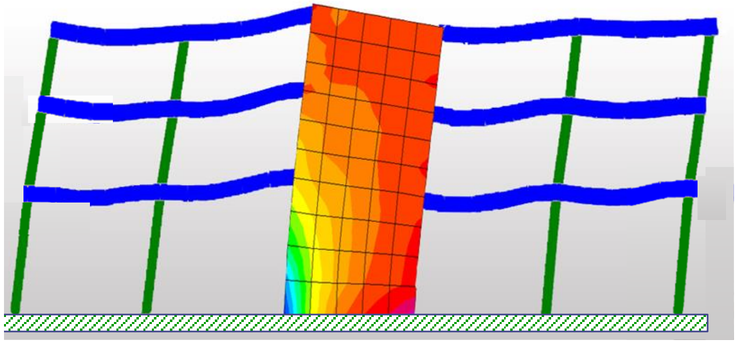
Structural Analysis

Outcome of Structural Analysis: Probability of each value (index i) of each EDP (index j) for each hazard level (index m): $\mathbf{p(EDP_j^i \,|\, Im_m)}$



Structural Analysis: Application

Analytical Simulation

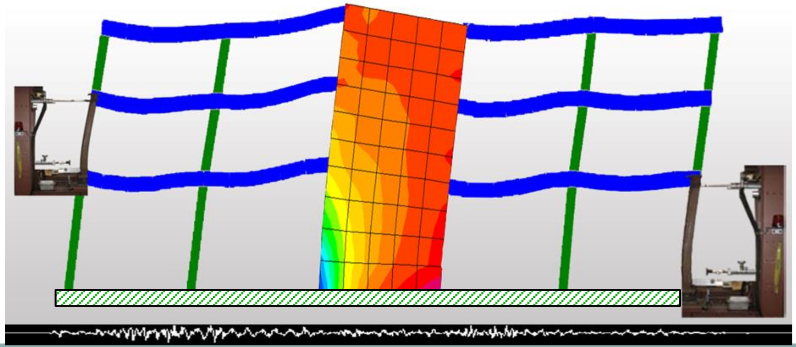


+

Experimental Simulation

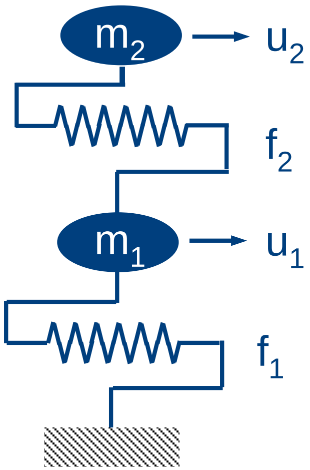
=

Hybrid Simulation



Structural Analysis: Application

Analytical Simulation:

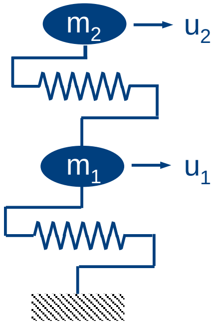


$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\mathbf{m}}\underbrace{\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}}_{\ddot{\mathbf{u}}} + \underbrace{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}}_{\mathbf{c}}\underbrace{\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix}}_{\dot{\mathbf{u}}} + \underbrace{\begin{bmatrix} -f_2 + f_1 \\ f_2 \end{bmatrix}}_{\mathbf{f}} = \underbrace{-\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}\ddot{u}_g}_{\mathbf{p}}$$

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{f} = \mathbf{p}$$

Analytical Simulation: Solve equation of motion using numerical integration methods

Structural Analysis: Application



$$\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}}_{\mathbf{m}\ \ddot{\mathbf{u}}} + \underbrace{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix}}_{\mathbf{c}\ \dot{\mathbf{u}}} + \underbrace{\begin{bmatrix} -f_2 + f_1 \\ f_2 \end{bmatrix}}_{\mathbf{f}} = \underbrace{-\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}\ddot{u}_g}_{\mathbf{p}}$$

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{f} = \mathbf{p}$$

A straightforward integration application: Explicit Newmark Integration

1) Compute the displacements $\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta t\,\dot{\mathbf{u}}_i + \dfrac{(\Delta t)^2}{2}\ddot{\mathbf{u}}_i$

2) Compute the restoring forces $\mathbf{f}_{i+1}$ corresponding to $\mathbf{u}_{i+1}$

3) Compute the accelerations $[\mathbf{m} + \Delta t\,\gamma\,\mathbf{c}]\ddot{\mathbf{u}}_{i+1} = \mathbf{p}_{i+1} - \mathbf{f}_{i+1} - \mathbf{c}[\dot{\mathbf{u}}_i + \Delta t(1-\gamma)\ddot{\mathbf{u}}_i]$

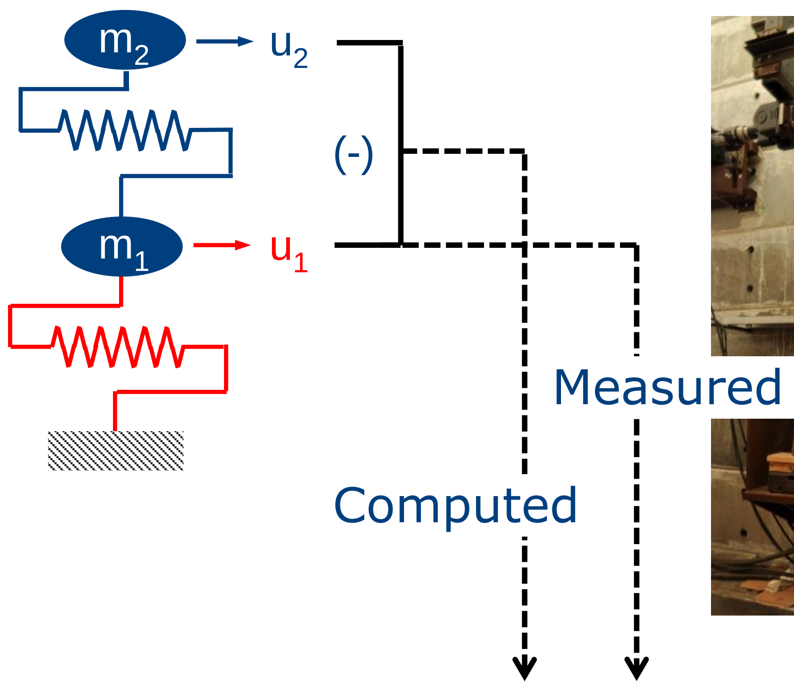
$$\mathbf{m}_{\mathbf{eff}}\ddot{\mathbf{u}}_{i+1} = \mathbf{p}_{\mathrm{eff}}$$

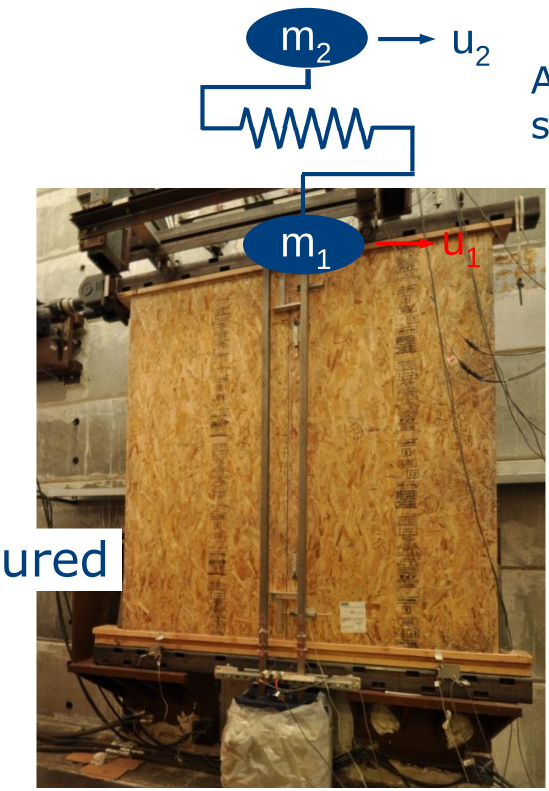
4) Compute the velocities $\dot{\mathbf{u}}_{i+1} = \dot{\mathbf{u}}_i + \Delta t[(1-\gamma)\ddot{\mathbf{u}}_i + \gamma\ddot{\mathbf{u}}_{i+1}]$

5) Increment *i*

Structural Analysis: Application

Bottom spring replaced as a test specimen



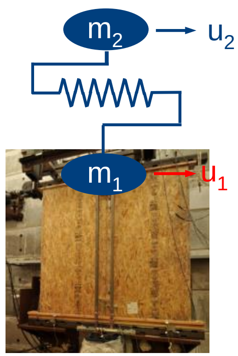


Analytical substructure

Experimental substructure

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}+\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix}+\begin{bmatrix} -f_a+f_e \\ f_a \end{bmatrix}=-\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}\ddot{u}_g$$

Structural Analysis: Application



$$\underbrace{\underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{\mathbf{m}}\underbrace{\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix}}_{\ddot{\mathbf{u}}} + \underbrace{\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}}_{\mathbf{c}}\underbrace{\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix}}_{\dot{\mathbf{u}}} + \underbrace{\begin{bmatrix} -f_a + f_e \\ f_a \end{bmatrix}}_{\mathbf{f}} = \underbrace{-\begin{bmatrix} m_1 \\ m_2 \end{bmatrix}\ddot{u}_g}_{\mathbf{p}}}_{\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{f} = \mathbf{p}}$$

A straightforward integration application: Explicit Newmark Integration

1) Compute the displacements $\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta t\,\dot{\mathbf{u}}_i + \frac{(\Delta t)^2}{2}\ddot{\mathbf{u}}_i$

2a) Compute the restoring force $f_{a,i+1}$ corresponding to the displacement $u_{2,i+1} - u_{1,i+1}$

2b) Impose $u_{1,i+1}$ to the test specimen and measure the corresponding force $f_{e,i+1}$

3) Compute the accelerations

$$[\mathbf{m} + \Delta t\,\gamma\,\mathbf{c}]\ddot{\mathbf{u}}_{i+1} = \mathbf{p}_{i+1} - \mathbf{f}_{i+1} - \mathbf{c}[\dot{\mathbf{u}}_i + \Delta t(1-\gamma)\ddot{\mathbf{u}}_i]$$

$$\mathbf{m}_{\mathbf{eff}}\ddot{\mathbf{u}}_{i+1} = \mathbf{p}_{\mathrm{eff}}$$

4) Compute the velocities $\dot{\mathbf{u}}_{i+1} = \dot{\mathbf{u}}_i + \Delta t[(1-\gamma)\ddot{\mathbf{u}}_i + \gamma\ddot{\mathbf{u}}_{i+1}]$

5) Increment *i*

Structural Analysis: Application

Why Hybrid Simulation?

IEEE693 requires seismic qualification of disconnect switches by shaking table tests, i.e. a switch & its support structure should be constructed, mounted to a shaking table & tested.

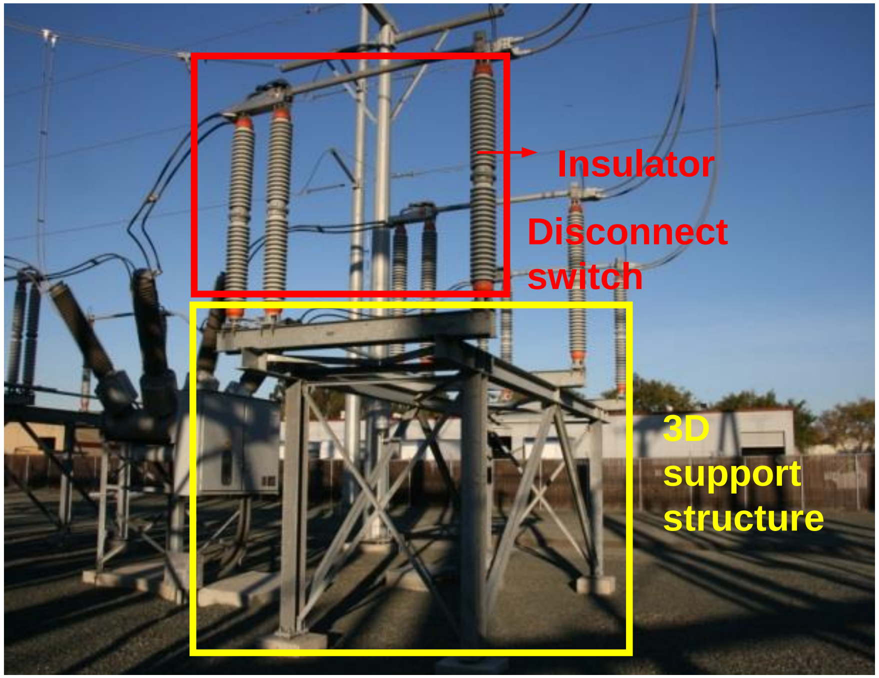
Structural Analysis: Application

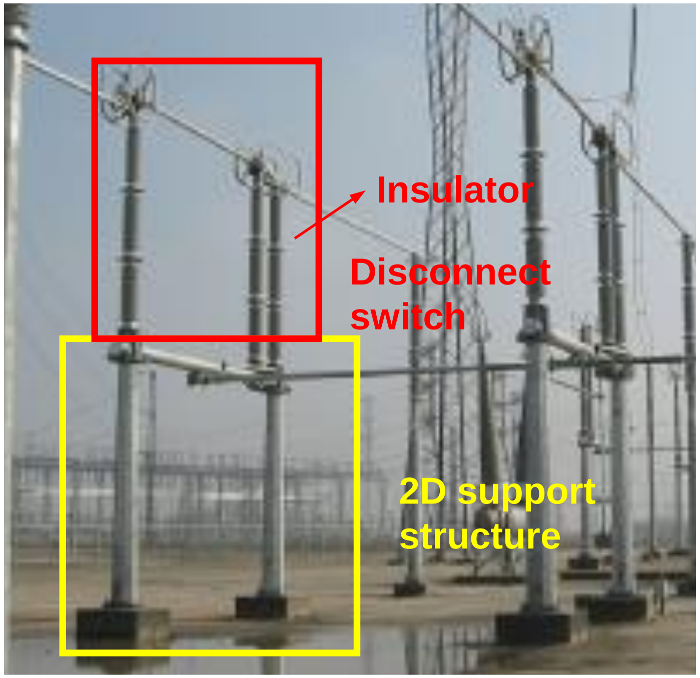
Why Hybrid Simulation?

Several tested configurations of 500 kV switch

Structural Analysis: Application

Why Hybrid Simulation?





Courtesy of Eric Fujisaki, PG&E

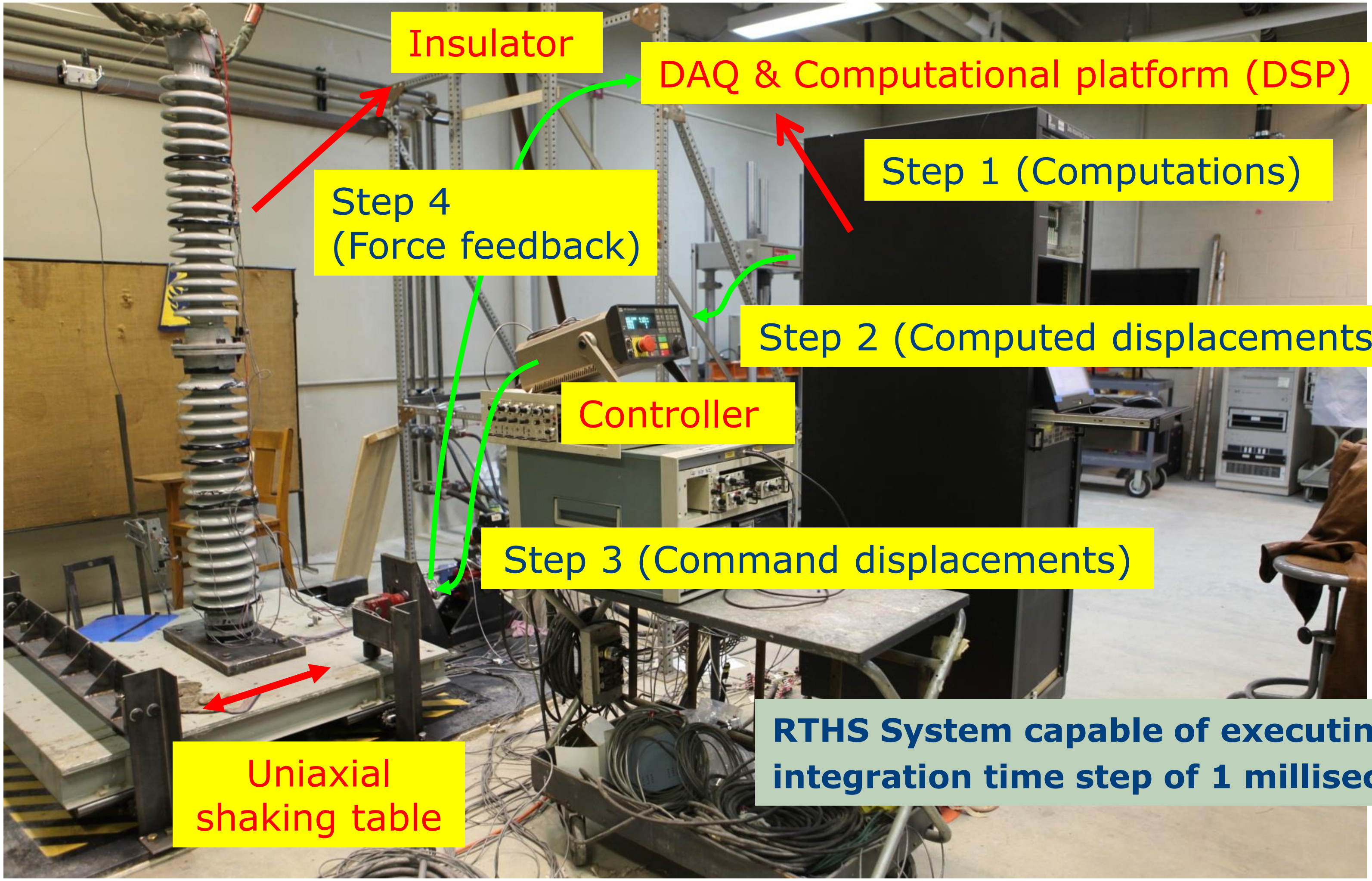
- ❑ Dynamic properties of support structures have major effect on response of switches.
- ❑ Several support structure configurations may need to be constructed until switch qualifies.
- ❑ A series of conventional shaking table tests is time-consuming & economically unfeasible.

Structural Analysis: Application

Why Hybrid Simulation?

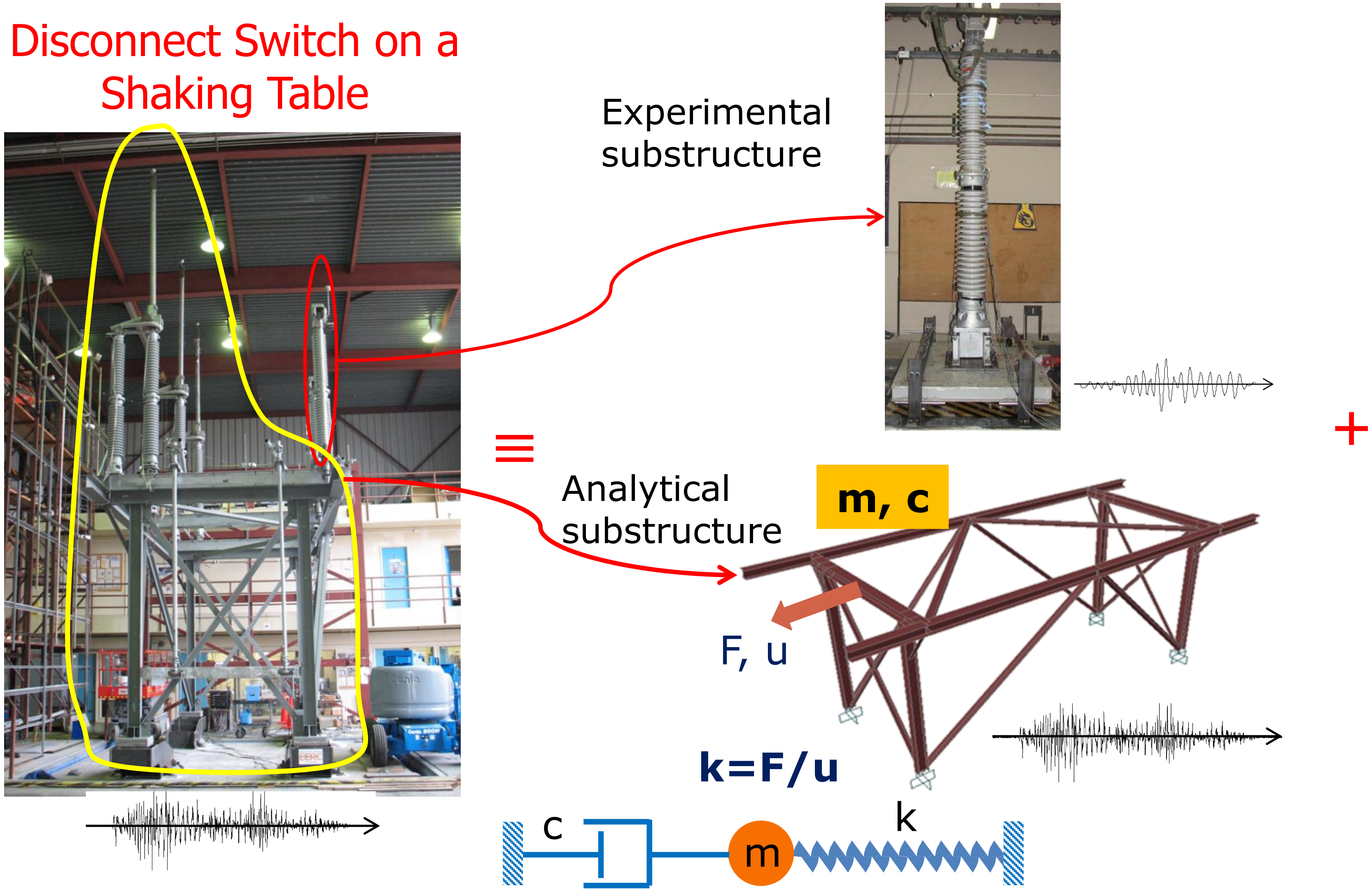
- Hybrid simulation (HS): a cost effective and efficient alternative to the conventional shaking table testing of disconnect switches.
- Real-time HS: Rate-dependent nature of some types of insulator posts, e.g. polymer composite ones, requires real-time HS (RTHS)
- HS on shaking table configuration: Distributed mass of insulator posts limits practical use of actuators at discrete locations along their height
- A RTHS system is developed for testing insulator posts of disconnect switches on a “smart” shaking table

Structural Analysis: Application



RTHS System capable of executing an integration time step of 1 millisecond

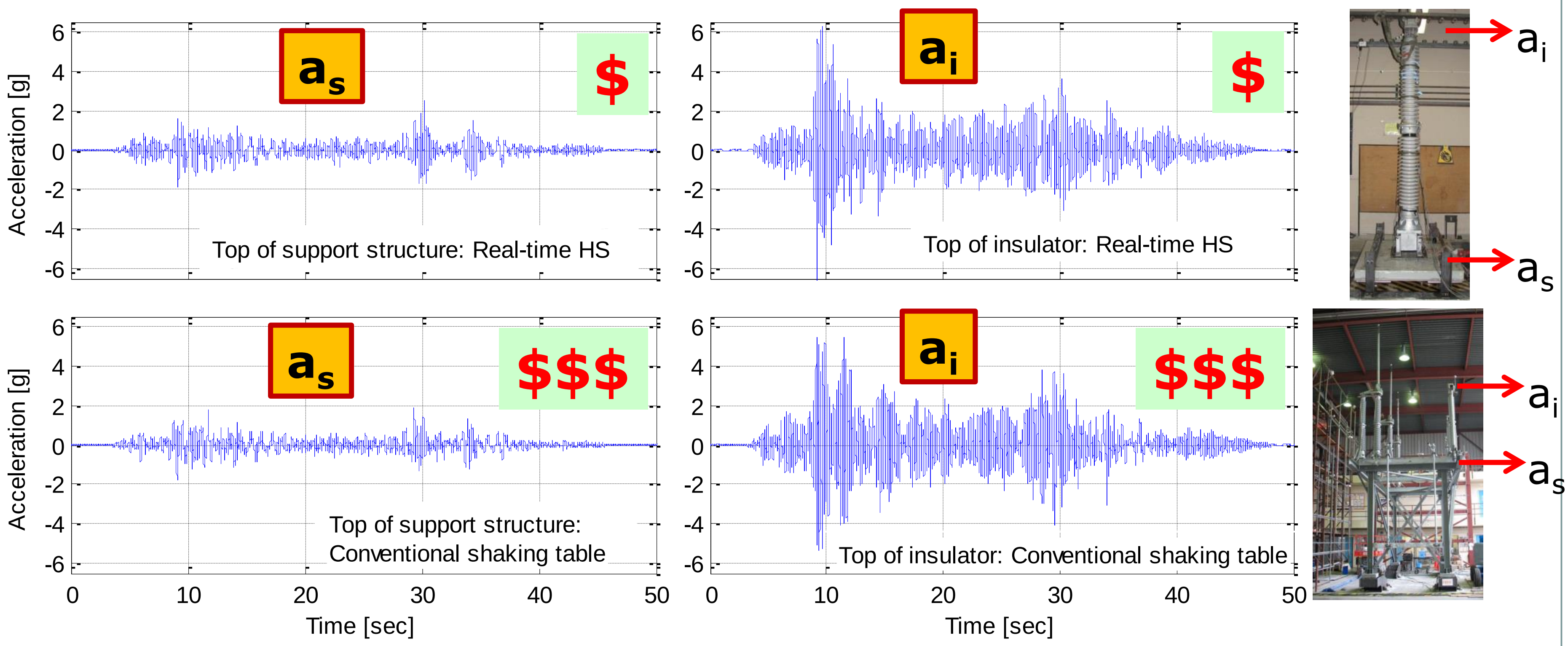
Structural Analysis: Application



Structural Analysis: Application

Comparison with conventional shaking table tests

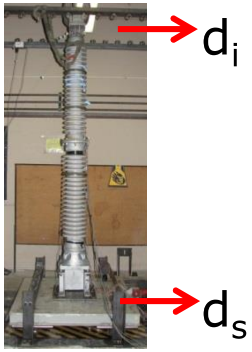
Accelerations at Top of Support Structure ($\mathbf{a_s}$) and Top of Insulator ($\mathbf{a_i}$)

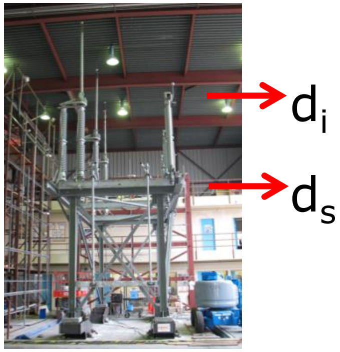


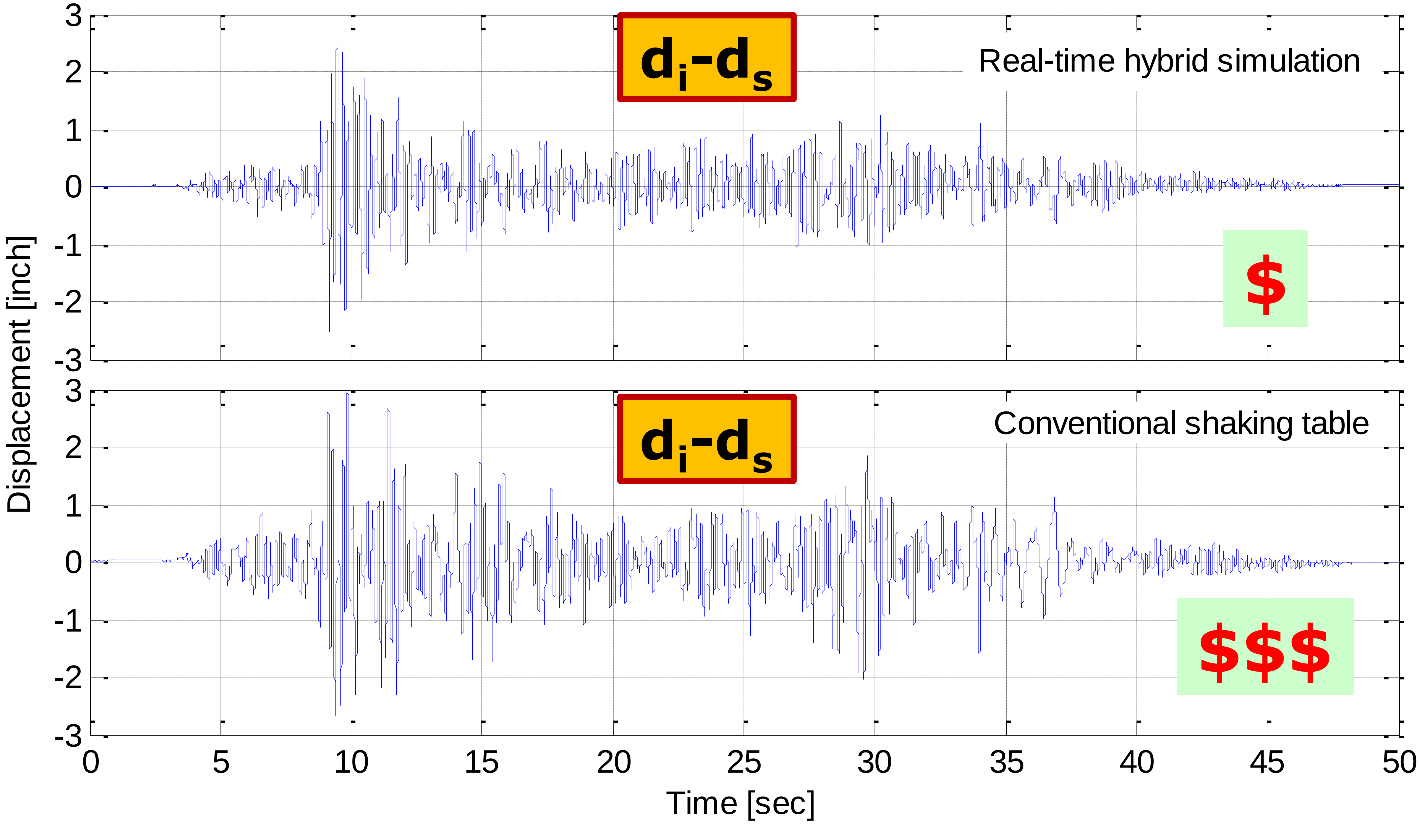
Structural Analysis: Application

Comparison with conventional shaking table tests

Relative displacement of insulator top w.r.t. top of support structure (d_i-d_s)







Structural Analysis: Application

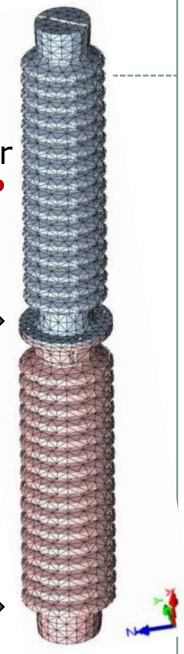
Parametric Study

Porcelain/Polymer Material Models ?

Interface ?

FE Model

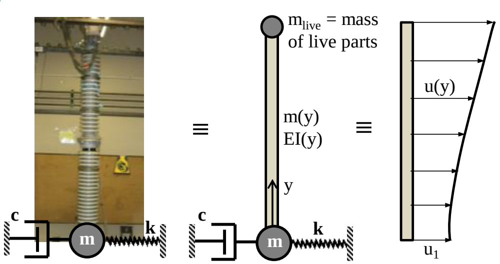
Boundary Conditions ?



Polymer

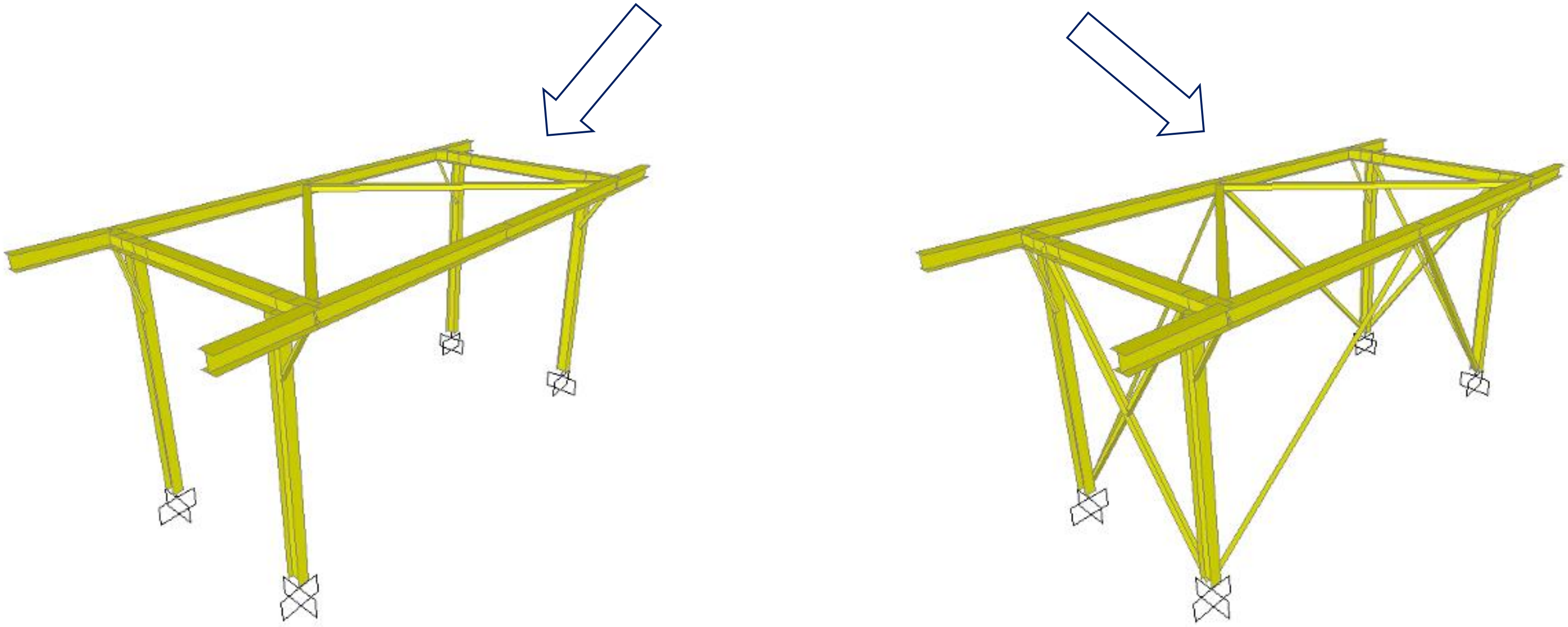
Porcelain

Distributed Mass → RTHS in Shaking Table Configuration



Structural Analysis: Application

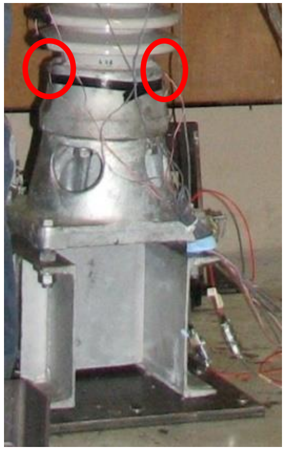
- 13 support structure stiffness, k, values between 2.2 kips/in and 60 kips/in

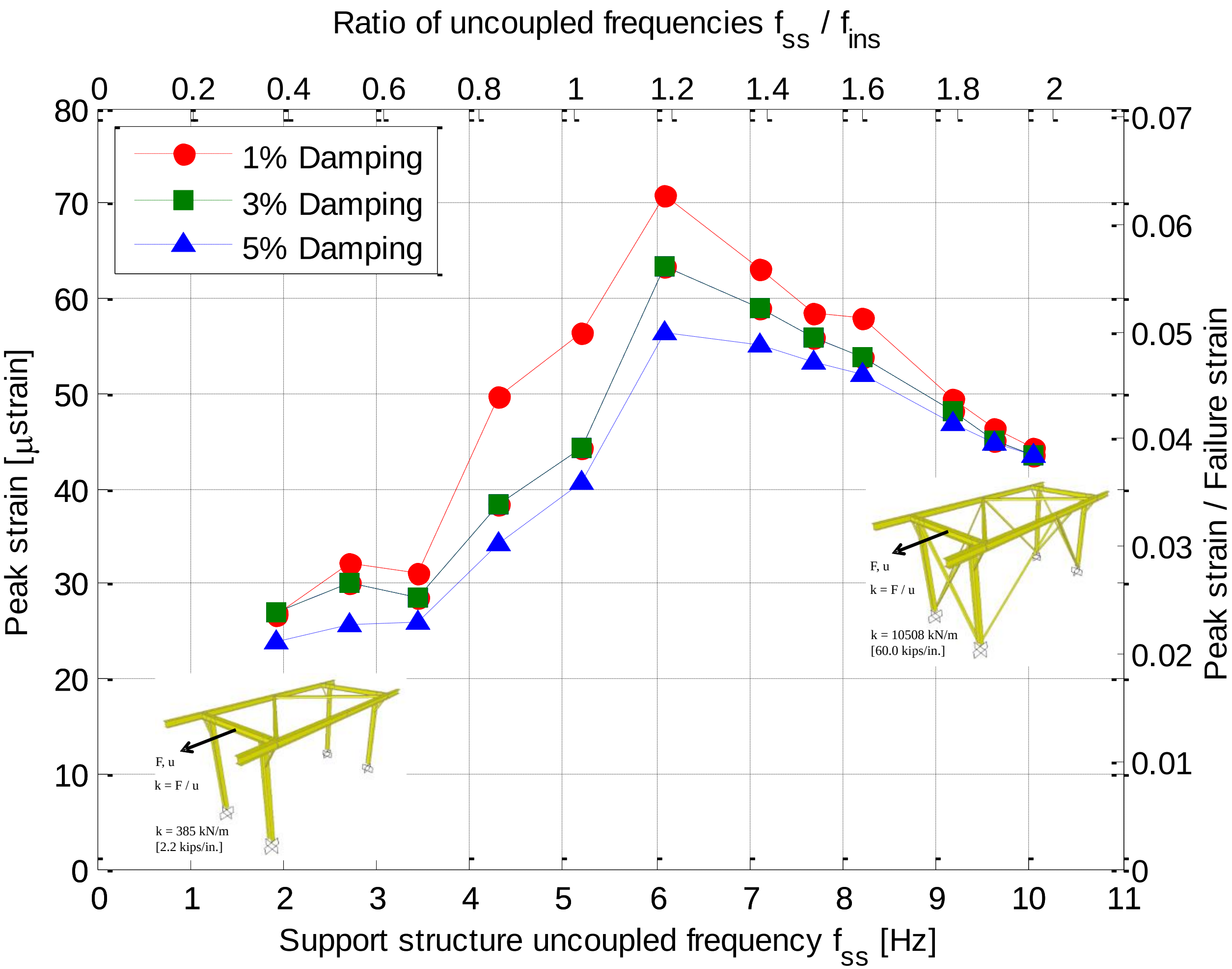


- 3 damping ratios for support structure: ξ= 1%, 3%, 5%
- 10% scale IEEE compatible ground motion

Structural Analysis: Application

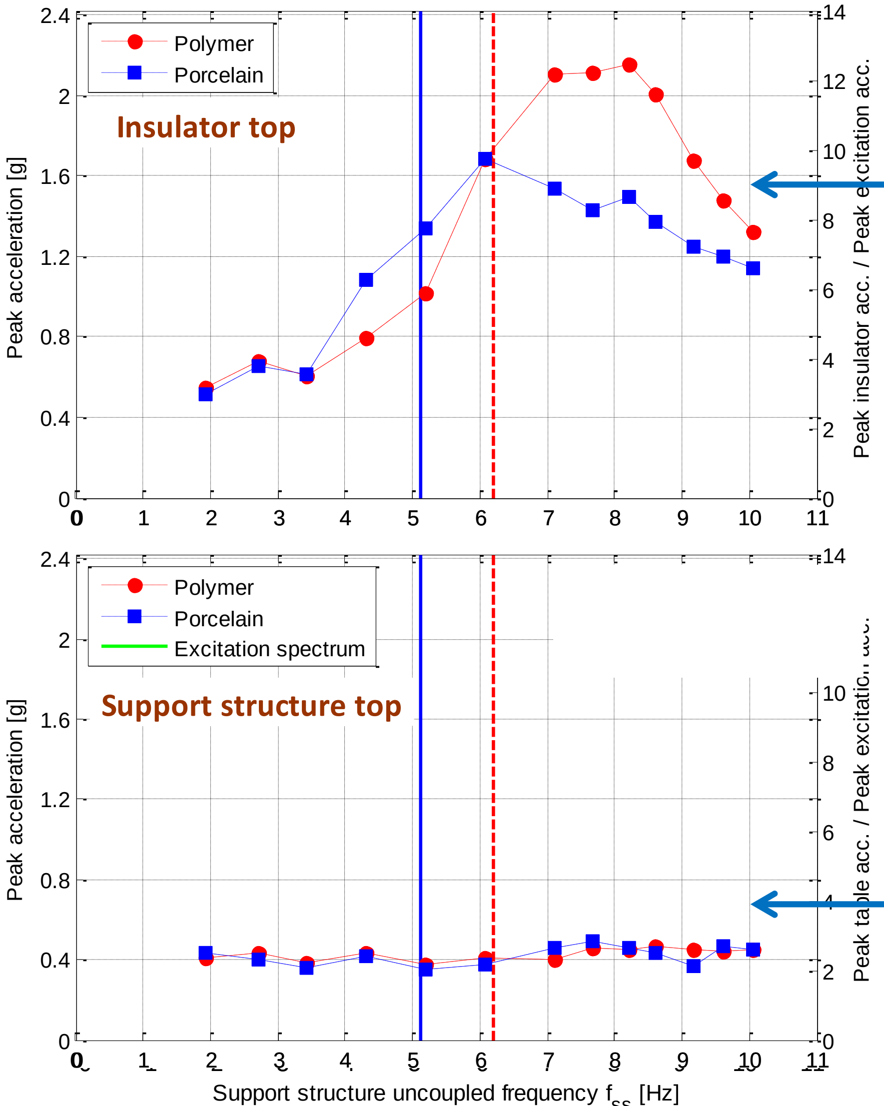
Strains: Porcelain Insulator



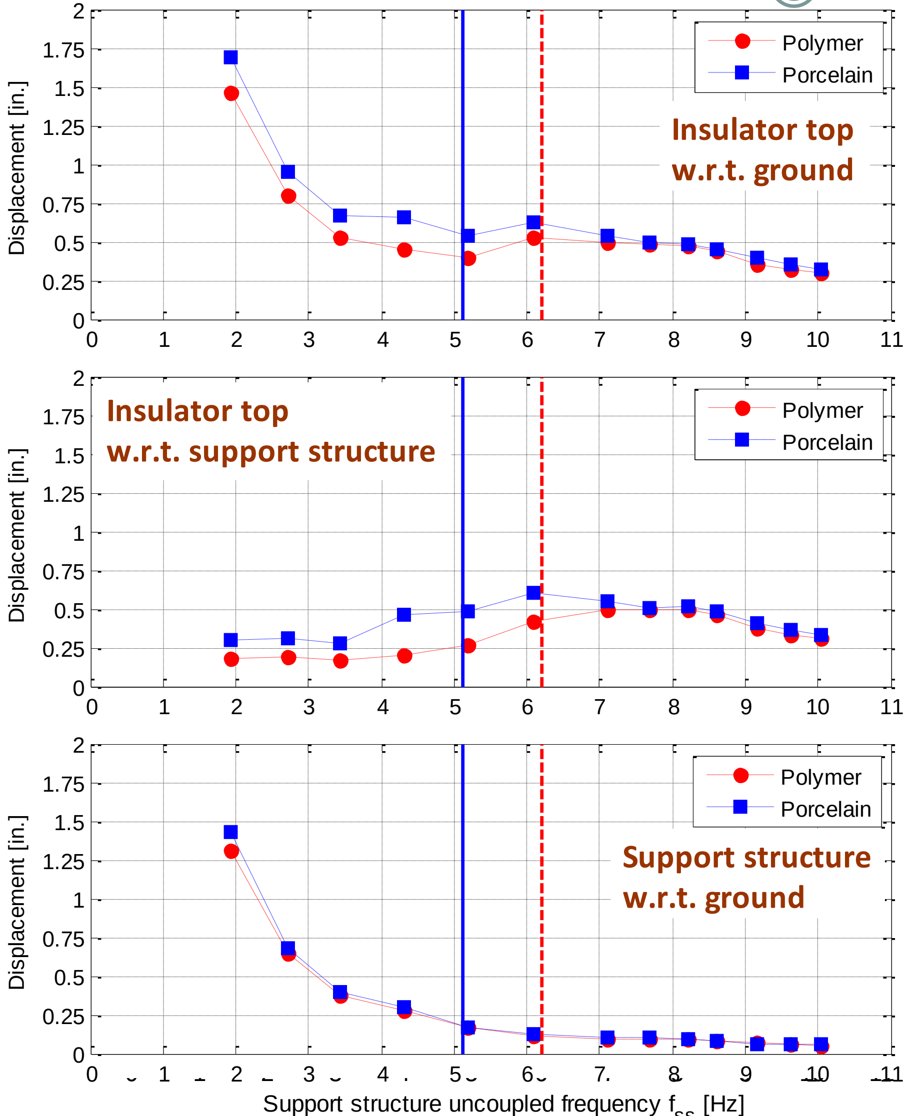


Structural Analysis: Application

Accelerations

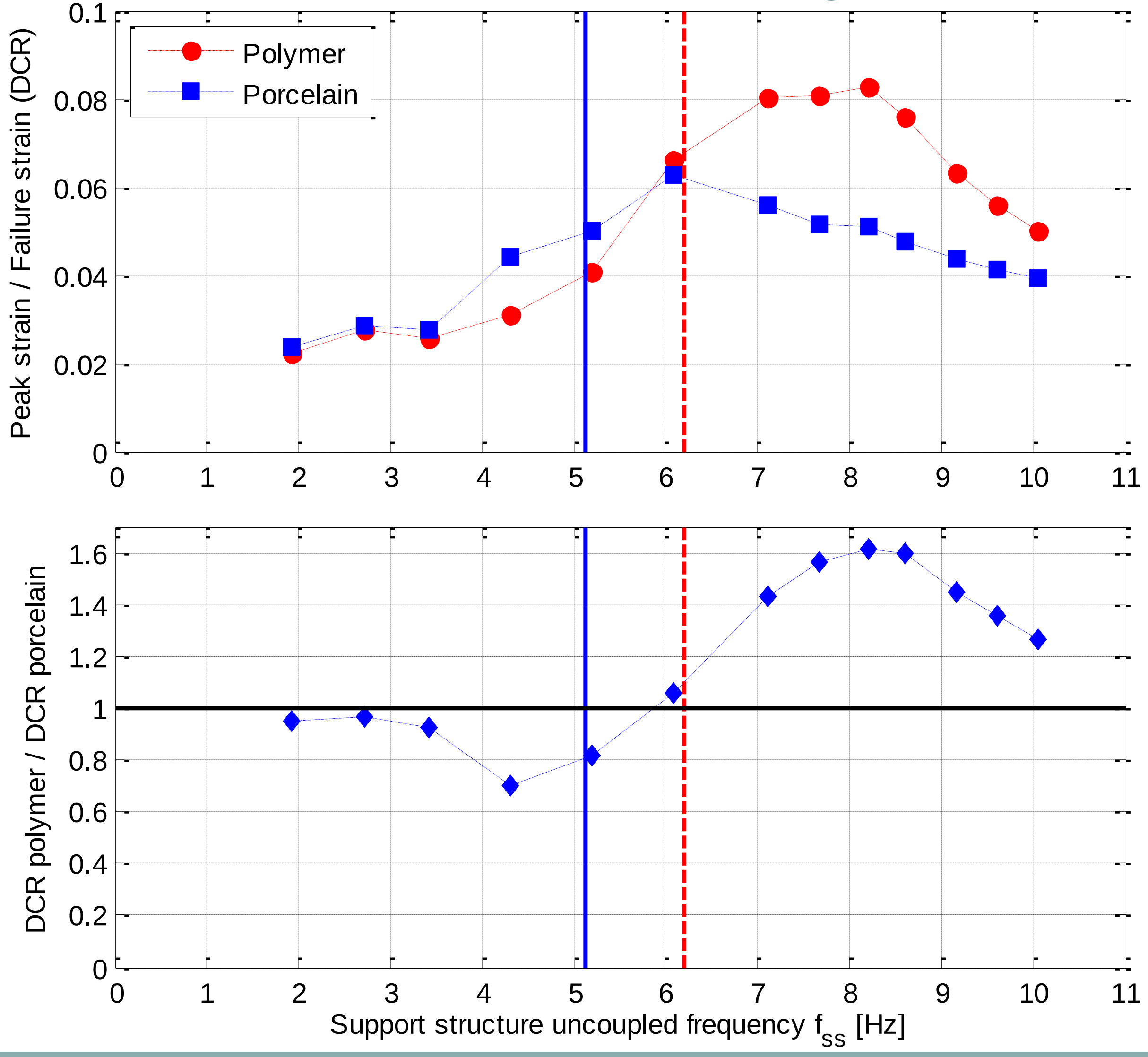


Structural Analysis: Application



Displacements

Structural Analysis: Application



Normalized Strains

Failure strains:

Polymer: **4800 μstrain**

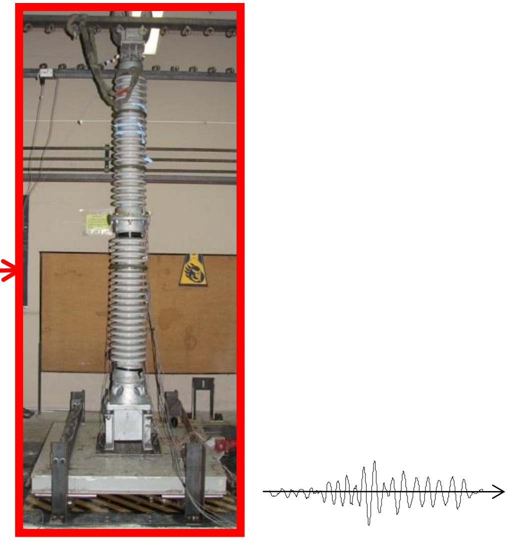
Porcelain: **1130 μstrain**

Structural Analysis: Application

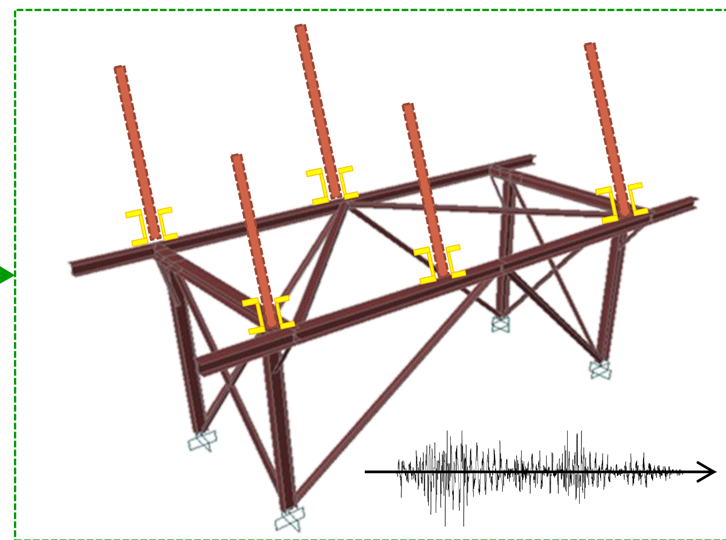
Conventional Shaking Table

≡

Experimental substructure

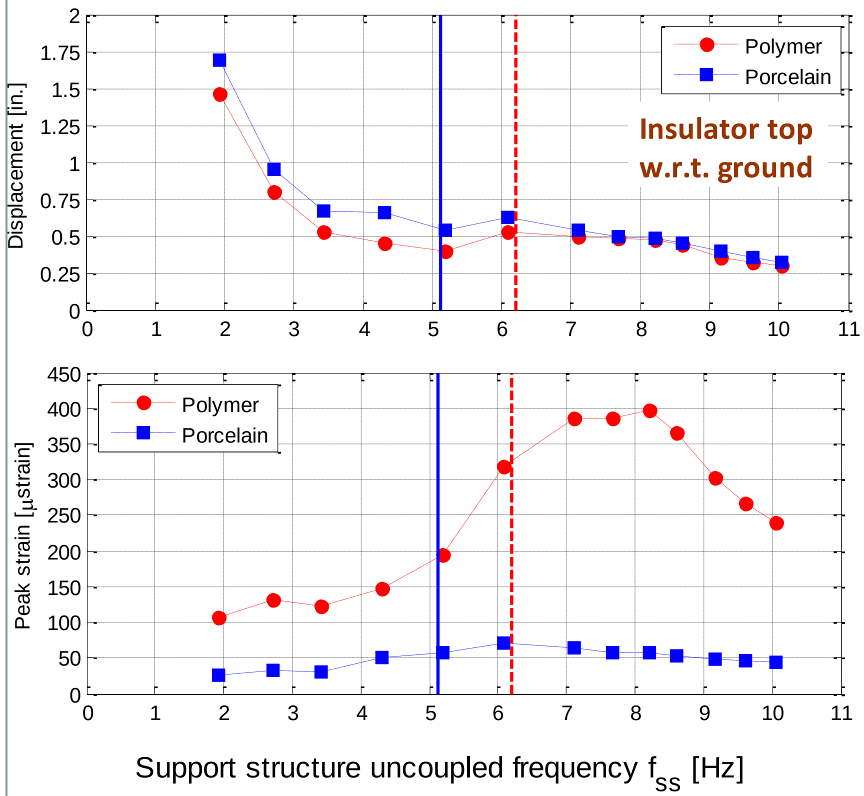


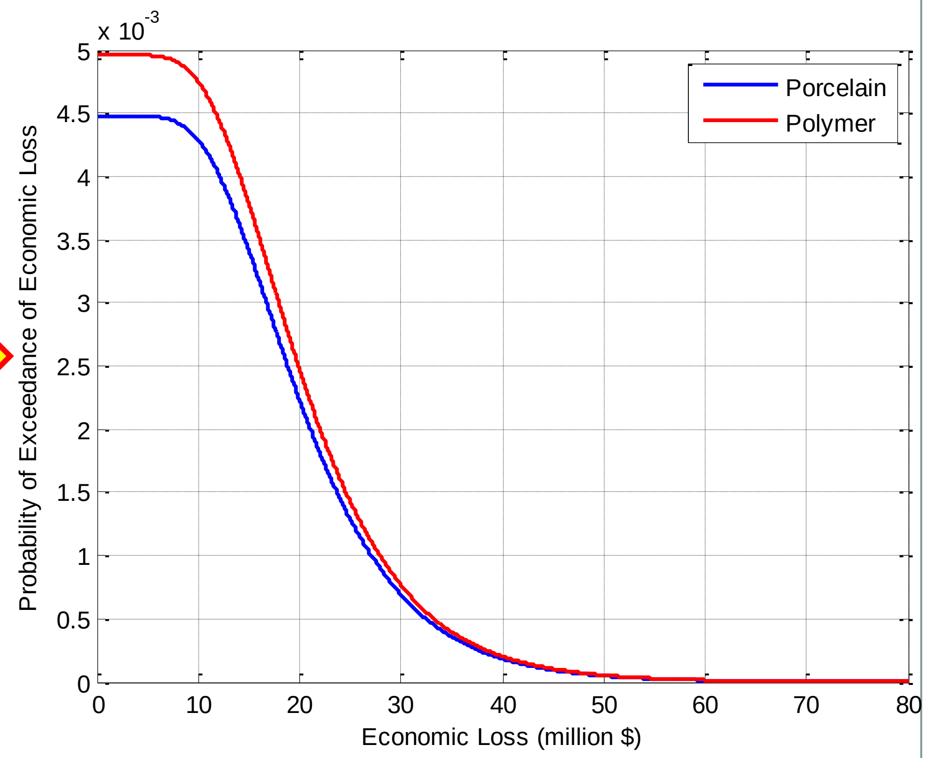
Analytical substructure



+ Corresponding RTHS

Structural Analysis: Application





Engineering Demand Parameter (EDP)
Meaningful to engineers

Decision Variable (DV)
Meaningful to all interested stakeholders

Structural Analysis: Application

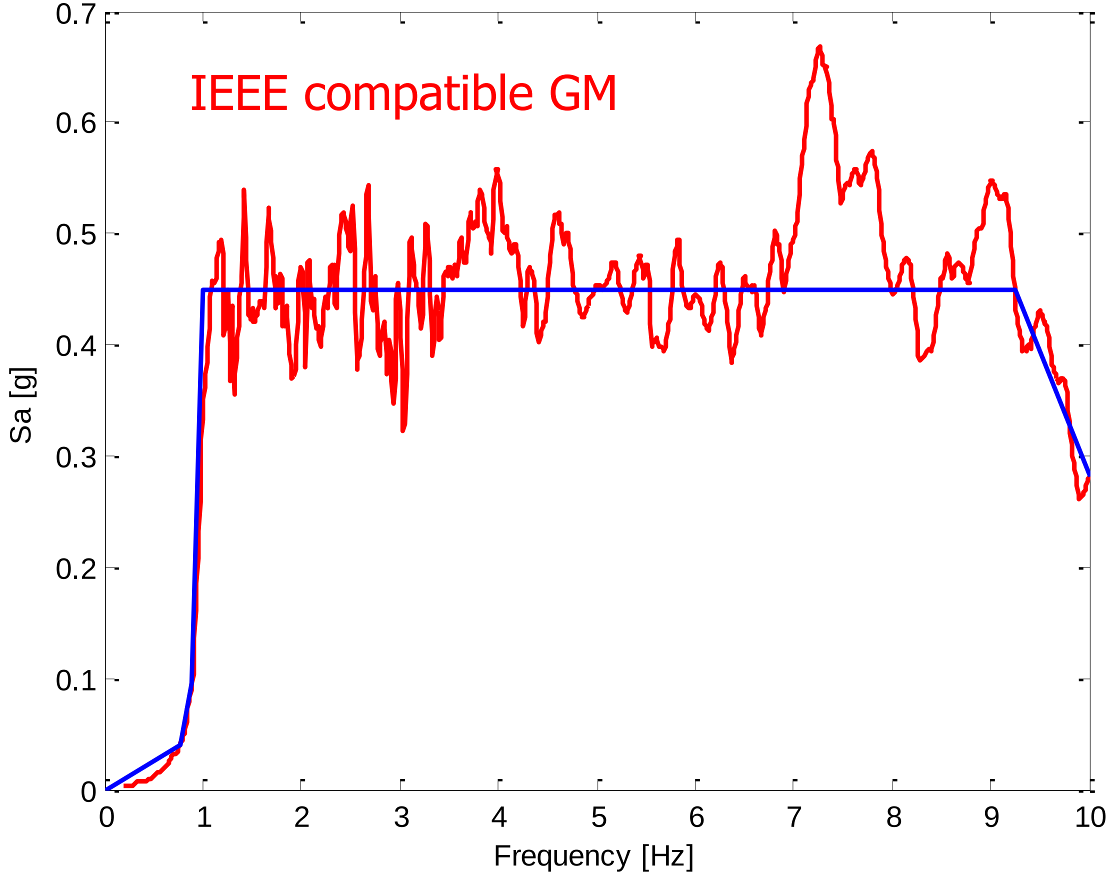
- Conventionally analytical simulation results
- Hybrid simulation results as another alternative in some applications

Two damageable groups and two EDPs

1) Insulator: Strain

2) Bus Connection: Displacement

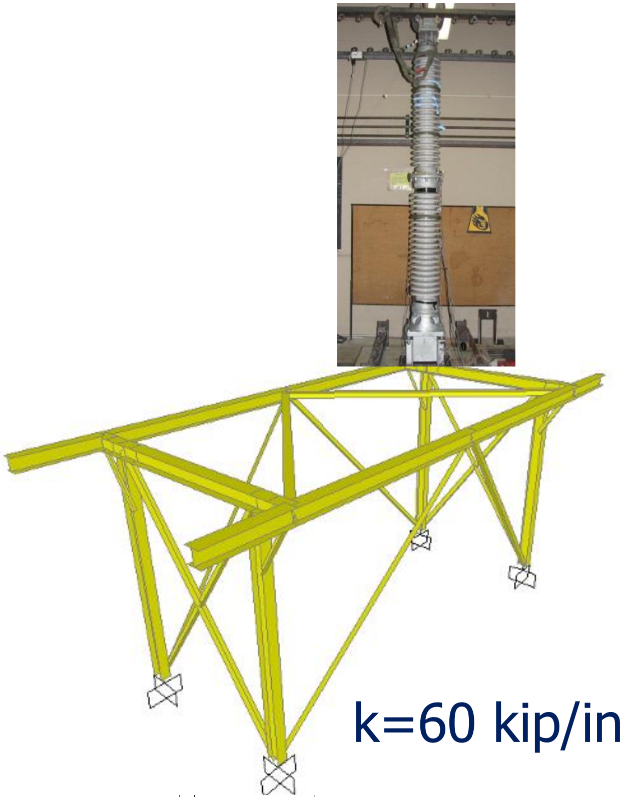
Structural Analysis: Application

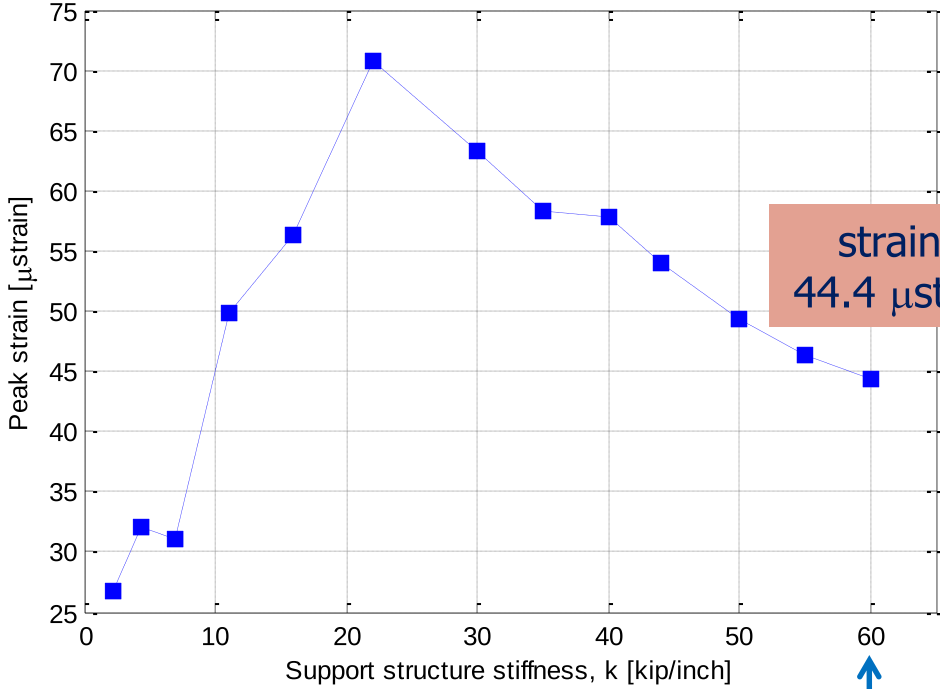


- Spectral acceleration of IEEE GM is constant between 1 and 9 Hz
- Response to IEEE GM is assumed to be median

Structural Analysis: Application

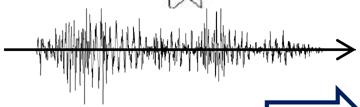
Structural Analysis: Porcelain Insulator on a Support Structure with Braces





strain=
44.4 μstrain

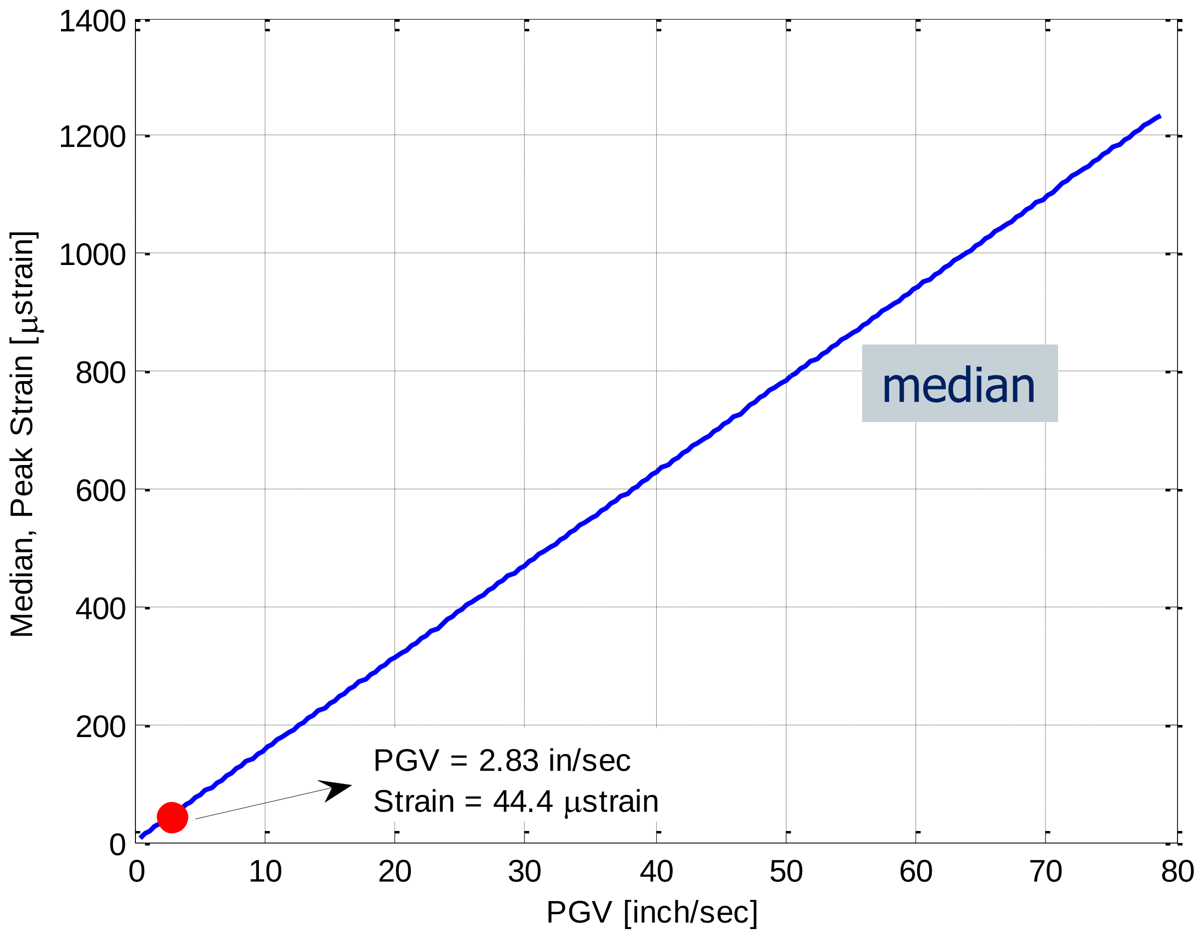
k=60 kip/inch



PGV=2.83 inch/sec

⇨ strain = 44.4 μstrain

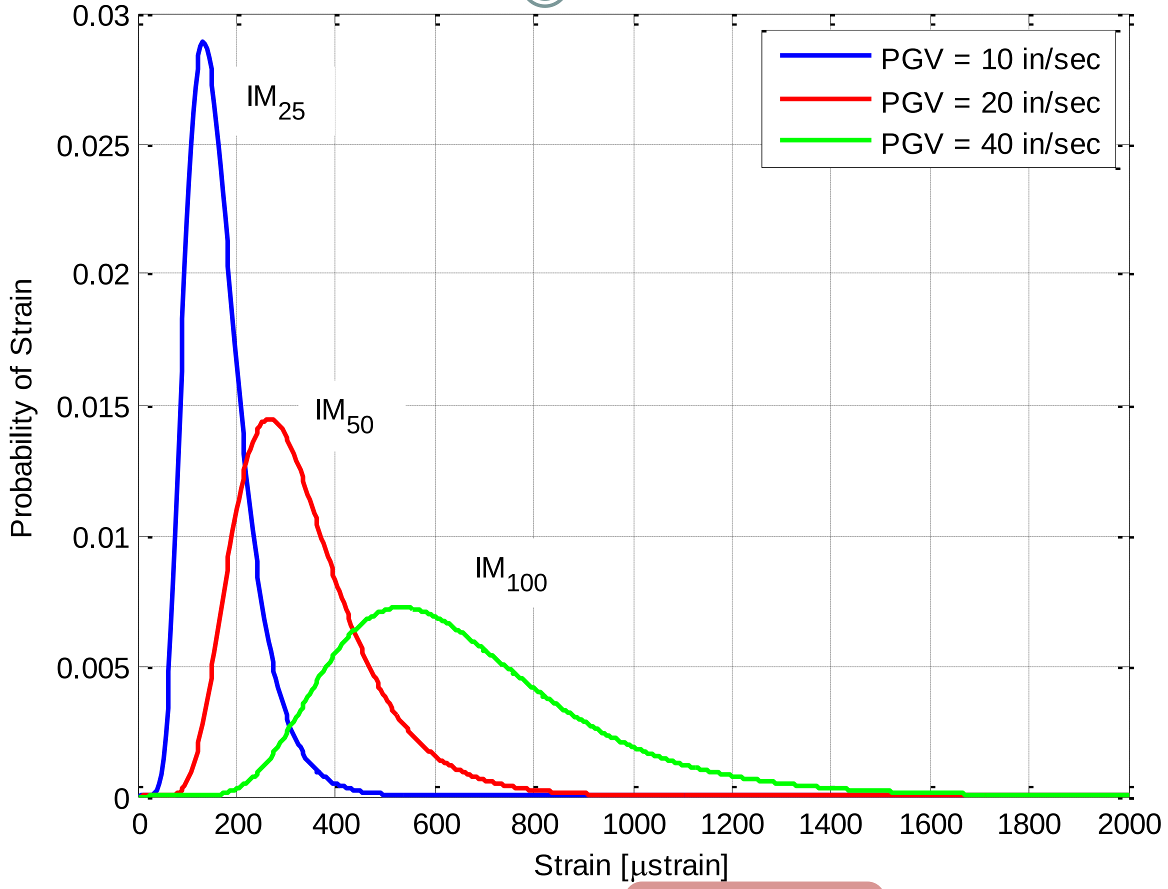
Structural Analysis: Application



- Coefficient of Variation (COV) is accepted to be 0.4

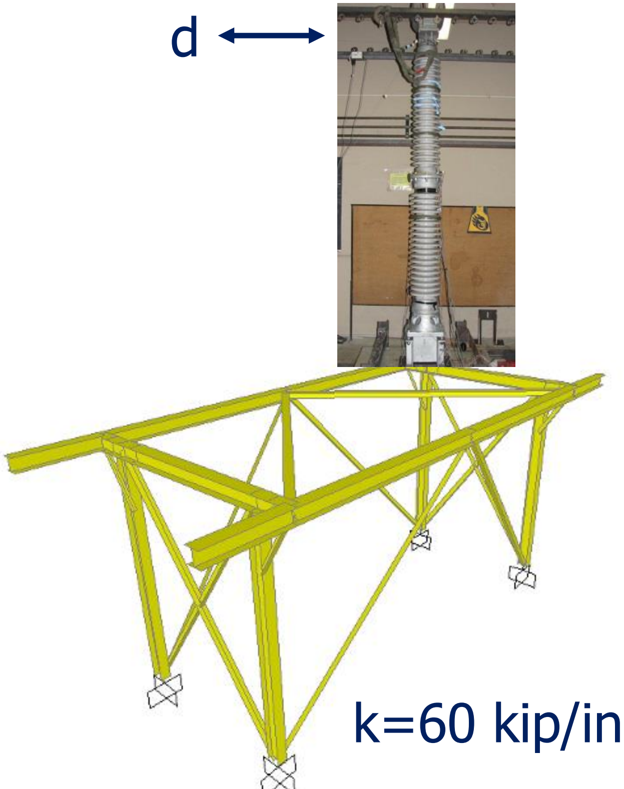
Structural Analysis: Application

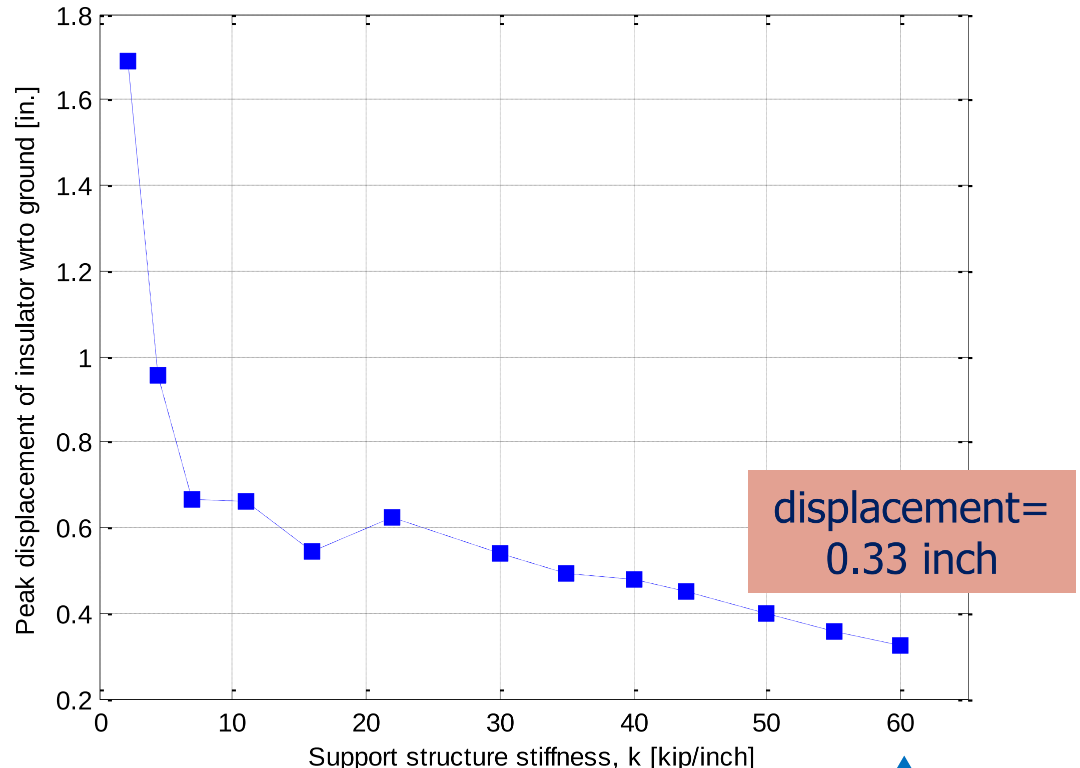
Strain
j=1



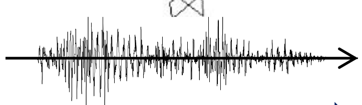
$$P\left(DV^{n}\right)=\sum_{m}\sum_{j}\sum_{i}\sum_{k}P\left(DV_{j}^{n}\middle|DM_{k}\right)p\left(DM_{k}\middle|EDP_{j}^{i}\right)\boxed{p\left(EDP_{j}^{i}\middle|IM_{m}\right)}p\left(IM_{m}\right)$$

Structural Analysis: Application





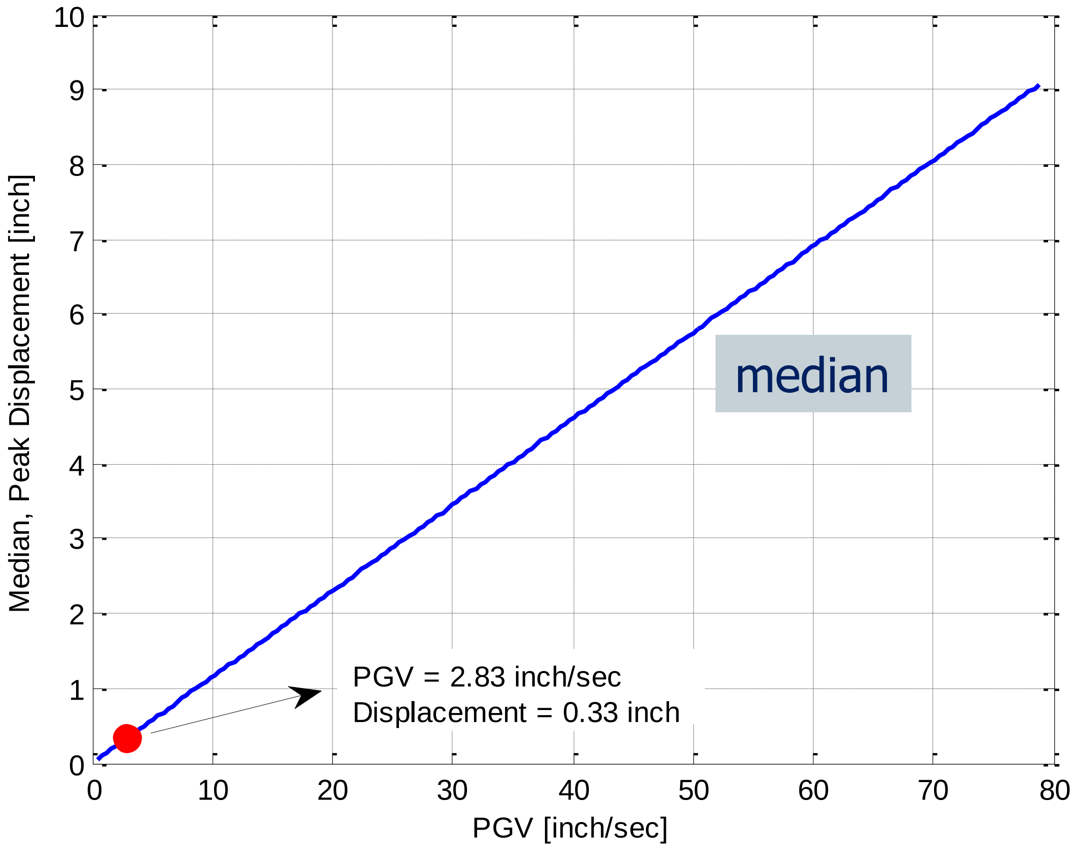
k=60 kip/inch



PGV=2.83 kip/inch

⇨ displacement = 0.33 inch

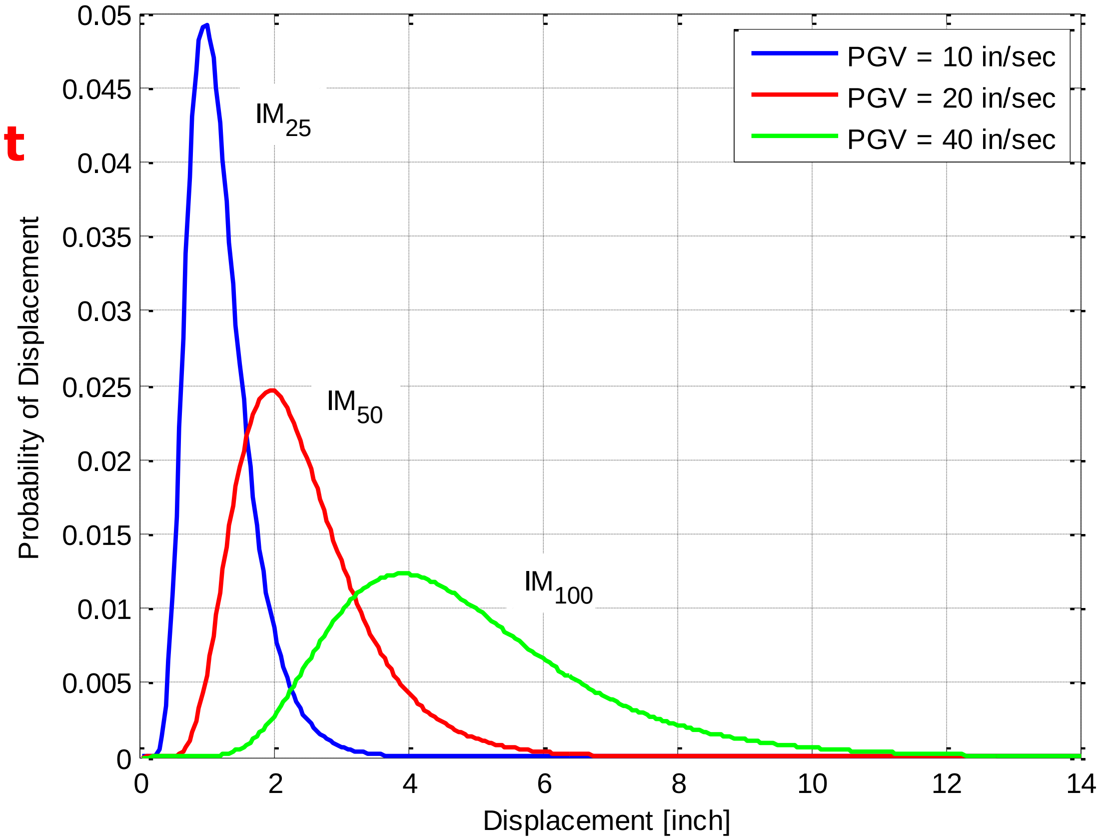
Structural Analysis: Application



- Coefficient of Variation (COV) is accepted to be 0.4

Structural Analysis: Application

Displacement
j=2



$$P\left(DV^n\right) = \sum_m \sum_j \sum_i \sum_k P\left(DV_j^n \middle| DM_k\right) p\left(DM_k \middle| EDP_j^i\right) \boxed{p\left(EDP_j^i \middle| IM_m\right)} p\left(IM_m\right)$$

Damage Analysis

- **PEER PBEE objective**: Performance definition in terms of the direct interest of not only engineers, but also various stakeholders
- **Damage analysis**: Third analysis stage to achieve this objective
- **Damage analysis objective**: Estimate physical damage (i.e. Damage Measure, **DM**) at the component or system levels as functions of the structural response
- **DMs**: Typically defined in terms of damage levels corresponding to repair measures to restore components of a facility to original conditions (other definitions are possible)
- **DM definition example**: Repair with epoxy injections (**light**); Repair with jacketing (**moderate**); Element replacement (**severe or collapse**)

Damage Analysis

- **Differences in path of achieving the same EDP**: A specific value of EDP corresponds to various DMs with different probabilities ← Uncertainty in damage analysis

Table 6-7 ***Modeling Parameters and Numerical Acceptance Criteria for Nonlinear Procedures— Reinforced Concrete Beams***

Conditions			Modeling Parameters[3]			Acceptance Criteria[3]				
			PR Plastic Rotation Angle, radians		Residual Strength Ratio	Plastic Rotation Angle, radians				
						Performance Level				
							Component Type			
							Primary		Secondary	
			a	b	c	IO	LS	CP	LS	CP
i. Beams controlled by flexure[1]										
$\frac{\rho - \rho'}{\rho_{bal}}$	Trans. Reinf.[2]	$\frac{V}{b_w d \sqrt{f'_c}}$								
≤ 0.0	C	≤ 3	0.025	0.05	0.2	0.010	0.02	0.025	0.02	0.05
≤ 0.0	C	≥ 6	0.02	0.04	0.2	0.005	0.01	0.02	0.02	0.04
≥ 0.5	C	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≥ 0.5	C	≥ 6	0.015	0.02	0.2	0.005	0.005	0.015	0.015	0.02
≤ 0.0	NC	≤ 3	0.02	0.03	0.2	0.005	0.01	0.02	0.02	0.03
≤ 0.0	NC	≥ 6	0.01	0.015	0.2	0.0015	0.005	0.01	0.01	0.015
≥ 0.5	NC	≤ 3	0.01	0.015	0.2	0.005	0.01	0.01	0.01	0.015
≥ 0.5	NC	≥ 6	0.005	0.01	0.2	0.0015	0.005	0.005	0.005	0.01

FEMA-356

- If PR≤0.01 → DM = IO
- If 0.01<PR≤0.02 → DM = LS
- If 0.02<PR≤0.025 → DM = CP

Examples:

- PR = 0.005 → DM = IO w/ p=100%
- PR = 0.015 → DM = LS w/ p=100%
- PR = 0.022 → DM = CP w/ p=100%
- PR = 0.030 → DM = Collapse w/ p=100%

Damage Analysis

FEMA-356

- PR = 0.005 → DM = IO with p=100%
- PR = 0.015 → DM = LS with p=100%
- PR = 0.022 → DM = CP with p=100%
- PR = 0.030 → DM = Collapse with p=100%

Note: Probability values are chosen arbitrarily for PEER-PBEE for illustration only.

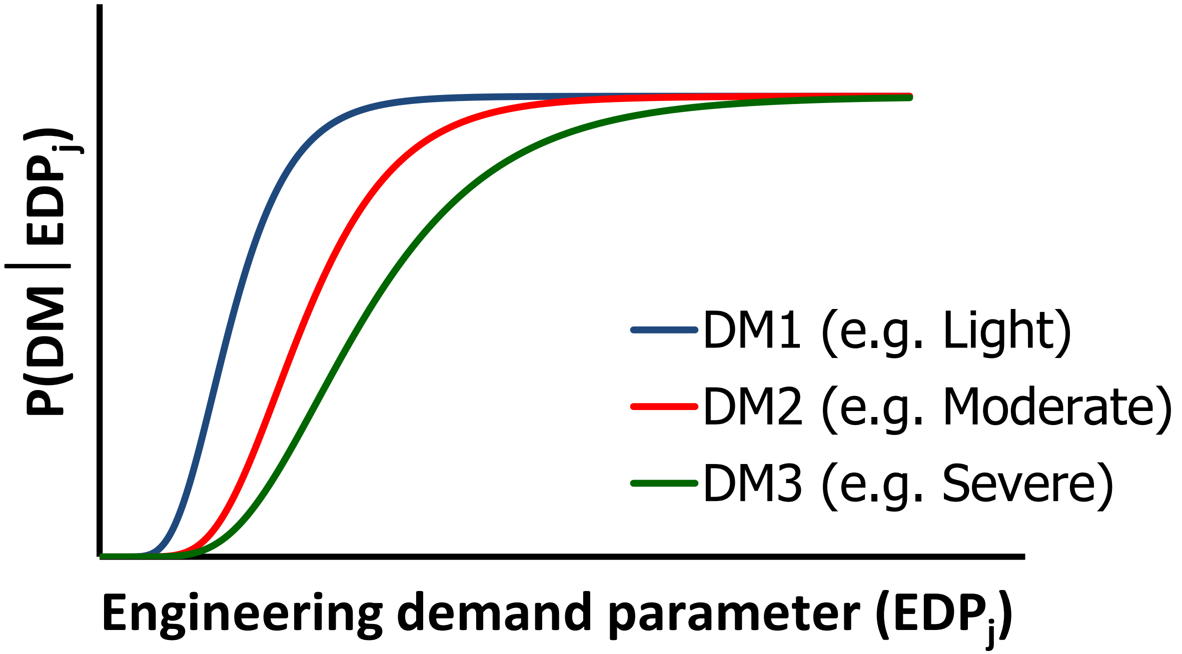
PEER-PBEE

- PR = 0.005 → DM = IO with p=70%, DM = LS with p=20%,
  DM = CP with p=8%, DM= collapse with p=2%
- PR = 0.015 → DM = IO with p=15%, DM = LS with p=60%,
  DM = CP with p=20%, DM= collapse with p=5%
- PR = 0.022 → DM = IO with p=5%, DM = LS with p=15%,
  DM = CP with p=60%, DM= collapse with p=20%
- PR = 0.030 → DM = IO with p=2%, DM = LS with p=12%,
  DM = CP with p=21%, DM= collapse with p=65%

Damage Analysis

- **Tool used in damage analysis**:

  Fragility function: POE of a DM for different values of an EDP

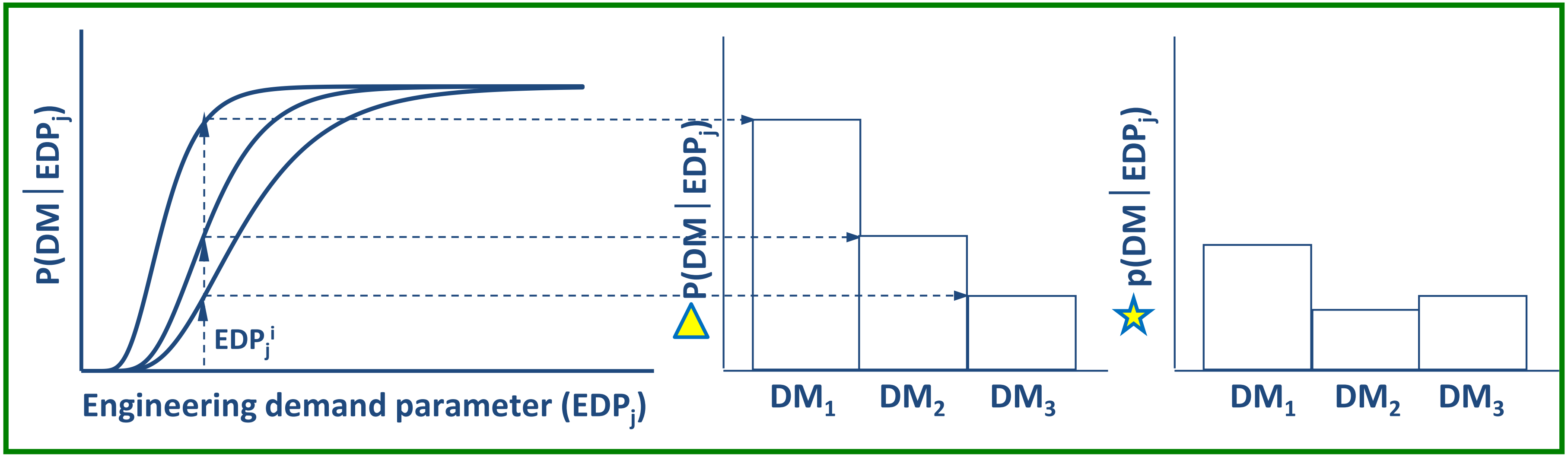


Damage Analysis

- Fragility function determination:
  - Analytical simulations
  - Experimental simulations (Hybrid simulation or Shaking table tests)
  - Generic functions based on expert opinion (not preferred)
- Damageable parts of a structure are divided into damageable groups:
  - Each damageable group consists of components that are affected by the same EDP in a similar way
  - The components in a group have the same fragility functions
  - **Example**: Bohl (2009) used **16** groups for a steel moment frame building: (1) structural system, (2) exterior enclosure, (3) drift-sensitive & (4) acceleration-sensitive non-structural elements & (5) office content for each floor

Damage Analysis

Outcome of Damage Analysis: Probability of each DM value (index k) for each value (index i) of each EDP (index j): $\mathbf{p(DM_k | EDP^i_j)}$



$$\text{for } k = 1 : \# \text{ of DM levels}$$

$$p(DM_k | EDP_j^i) = P(DM_k | EDP_j^i) \quad \text{if } k = \# \text{ of DM levels}$$

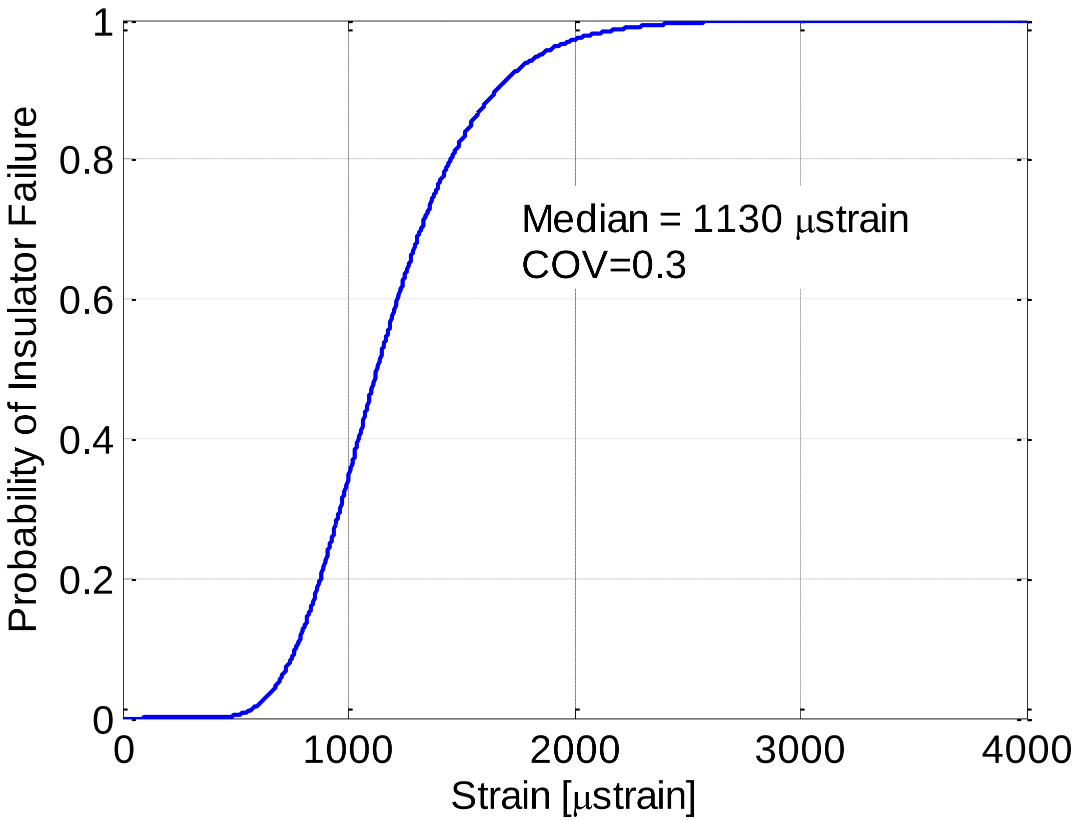
$$p(DM_k | EDP_j^i) = P(DM_k | EDP_j^i) - P(DM_{k+1} | EDP_j^i) \quad \text{otherwise}$$

Damage Analysis: Application

Damagable Group 1: Insulator

Damage State: Failure

Strain=Failure Strain



$$P\left(DV^{n}\right)=\sum_{m}\sum_{j}\sum_{i}\sum_{k}P\left(DV_{j}^{n}\middle|DM_{k}\right)p\left(DM_{k}\middle|EDP_{j}^{i}\right)p\left(EDP_{j}^{i}\middle|IM_{m}\right)p\left(IM_{m}\right)$$

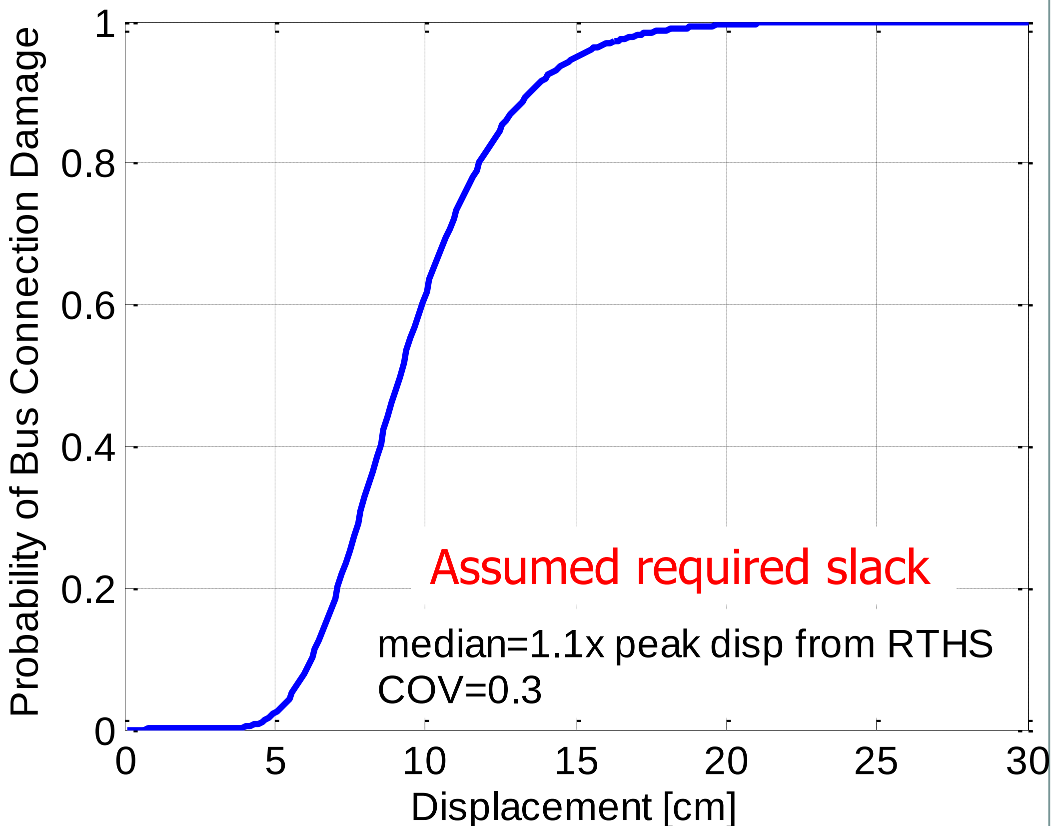
Damage Analysis: Application

Damagable Group 2: Bus connection

Damage State:

Bus Connection Damage ← Impact

Displacement=Slack



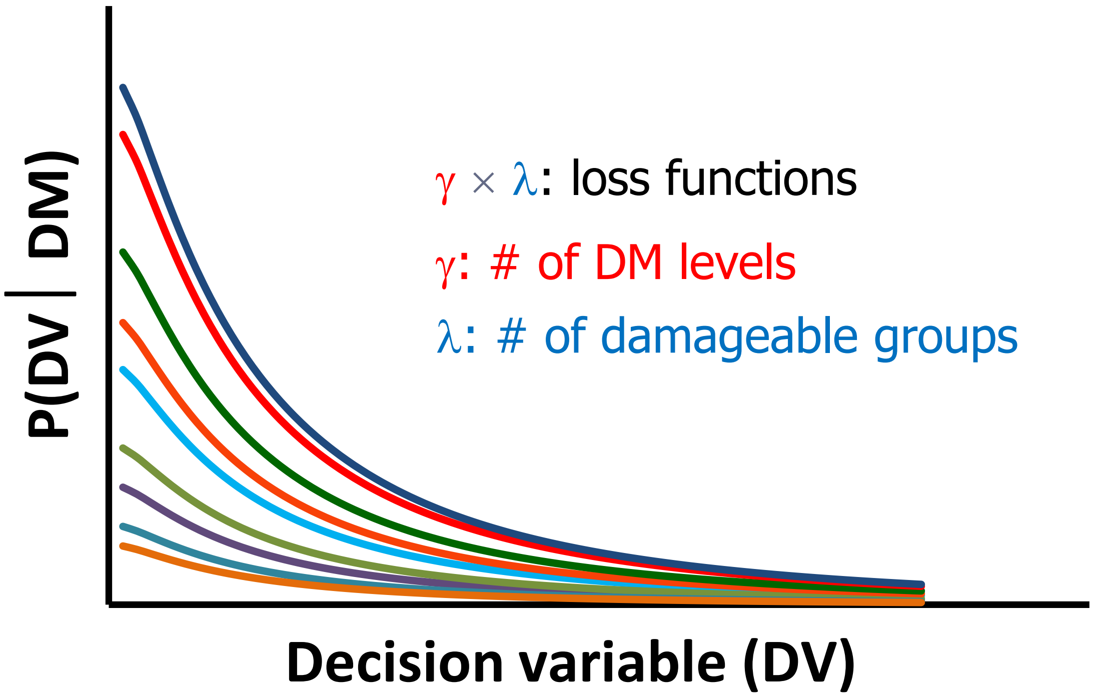
$$P\left(DV^{n}\right)=\sum_{m}\sum_{j}\sum_{i}\sum_{k}P\left(DV_{j}^{n}\middle|DM_{k}\right)\boxed{p\left(DM_{k}\middle|EDP_{j}^{i}\right)}\,p\left(EDP_{j}^{i}\middle|IM_{m}\right)p\left(IM_{m}\right)$$

Loss Analysis

- Last (Fourth) analysis stage in PEER PBEE Formulation
- **Damage information obtained from damage analysis**: Converted to the final decision variables (DVs)
- **Commonly utilized DVs**:
  - Fatalities
  - Economic loss
  - Repair duration
  - Injuries
- **Distribution of damage within the damageable group**: A specific value of DM corresponds to various DVs with different probabilities ← Uncertainty in loss analysis
- **Economic loss or repair cost as DV**: Uncertainty originating from the economical values, e.g. fluctuation in the market prices, is included

Loss Analysis

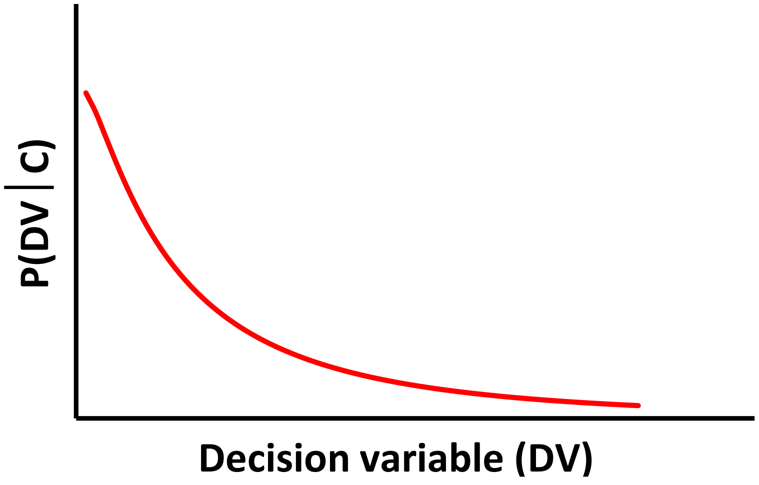
➢ **Tool used in loss analysis**:
Loss function: POE of a DV for different damageable groups and DMs



Loss Analysis

Loss function for collapse:

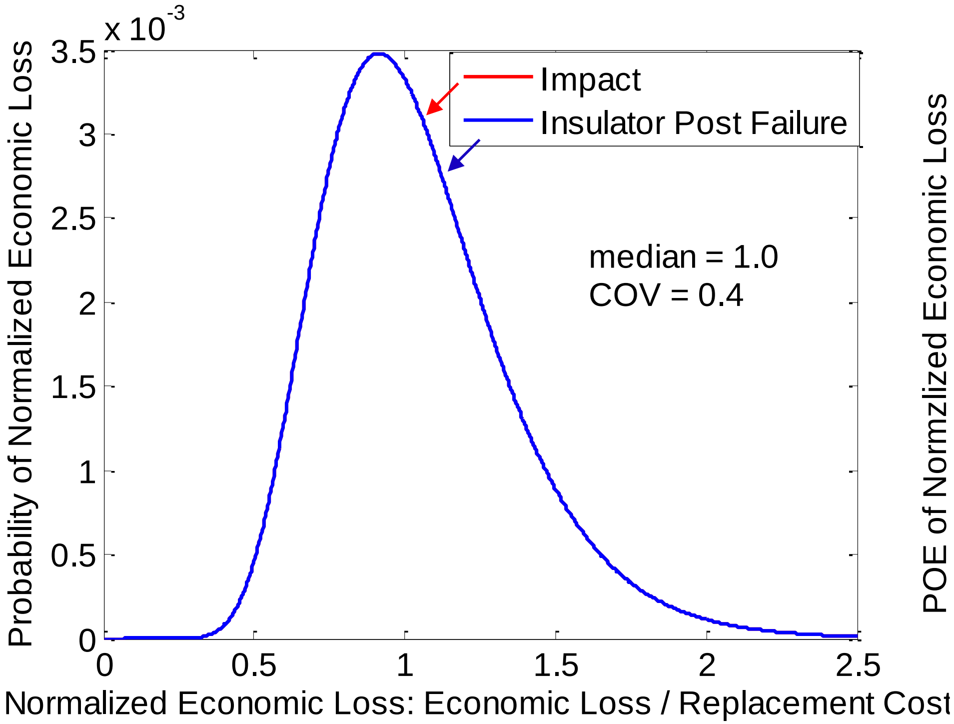
- Krawinkler (2005) assumed a lognormal distribution for P(DV|C)
- The expected value can be assumed as the total cost of the structural & nonstructural components of the facility
- **Following factors can be considered as sources of variance**:
  - ❖ Lack of information about all present structural & non-structural components
  - ❖ Lack of monetary value information about the components
  - ❖ Fluctuation in market prices

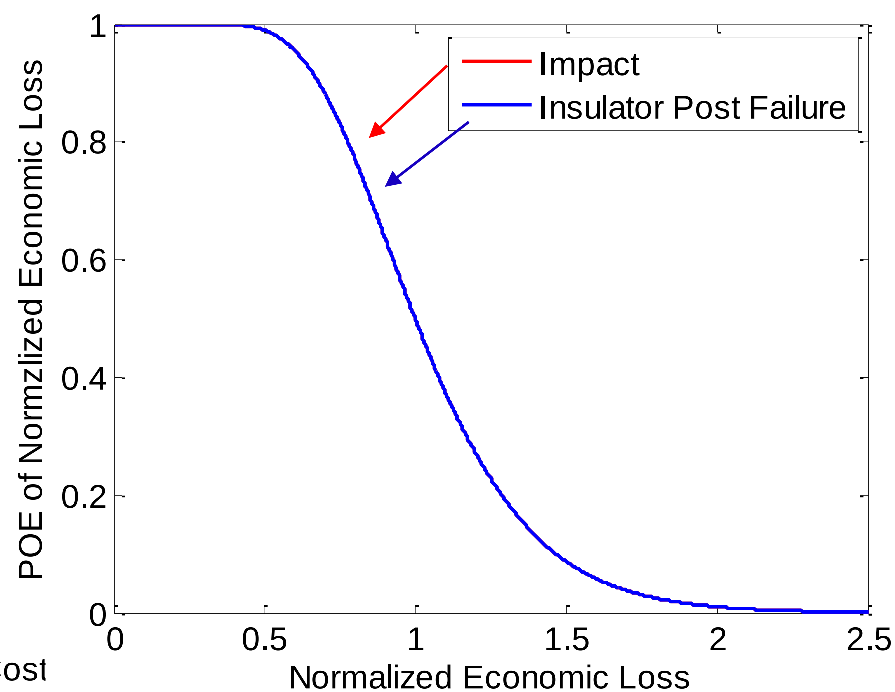


Loss Analysis: Application

- Only direct losses considered (to be updated in future)
- Both damage modes are accepted to result in replacement of the disconnect switch
- **DV**: Economic loss normalized with "unknown" replacement cost of a 230 kV switch

Loss Analysis: Application





$$P\left(DV^n\right) = \sum_m \sum_j \sum_i \sum_k P\left(DV_j^n \middle| DM_k\right) p\left(DM_k \middle| EDP_j^i\right) p\left(EDP_j^i \middle| IM_m\right) p\left(IM_m\right)$$

Combination of Analyses

Total probability theorem:

Given n mutually exclusive events* $A_1,\ldots, A_n$ whose probabilities sum to 1.0, then the probability of an arbitrary event B:

$$p(B) = p(B|A_1)\,p(A_1) + p(B|A_2)\,p(A_2) + \ldots + p(B|A_n)\,p(A_n)$$

$$p(B) = \sum_i \underbrace{p(B|A_i)}_{\text{Conditional probability of B given the presence of } A_i}\ \underbrace{p(A_i)}_{\text{Probability of } A_i}$$

*Occurrence of any one of them automatically implies the non-occurrence of the remaining n−1 events

Combination of Analyses

PEER PBEE combination of analyses: based on total probability theorem

Hazard Analysis

End product: $P(DV^n) = \sum_m P(DV^n | IM_m)\, p(IM_m)$

POE of the n^th value of the DV of the facility

Structural Analysis: Probability of no-collapse & of collapse

$$P(DV^n|IM_m) = P(DV^n|NC, IM_m)\, p(NC|IM_m) + P(DV^n|C)\, p(C|IM_m)$$

Loss Analysis: Loss function for collapse

$$P(DV^n|NC, IM_m) = \sum_j P(DV_j^n|NC, IM_m)$$

Structural Analysis

$$P(DV_j^n|NC, IM_m) = \sum_i P(DV_j^n|EDP_j^i)\, p(EDP_j^i|IM_m)$$

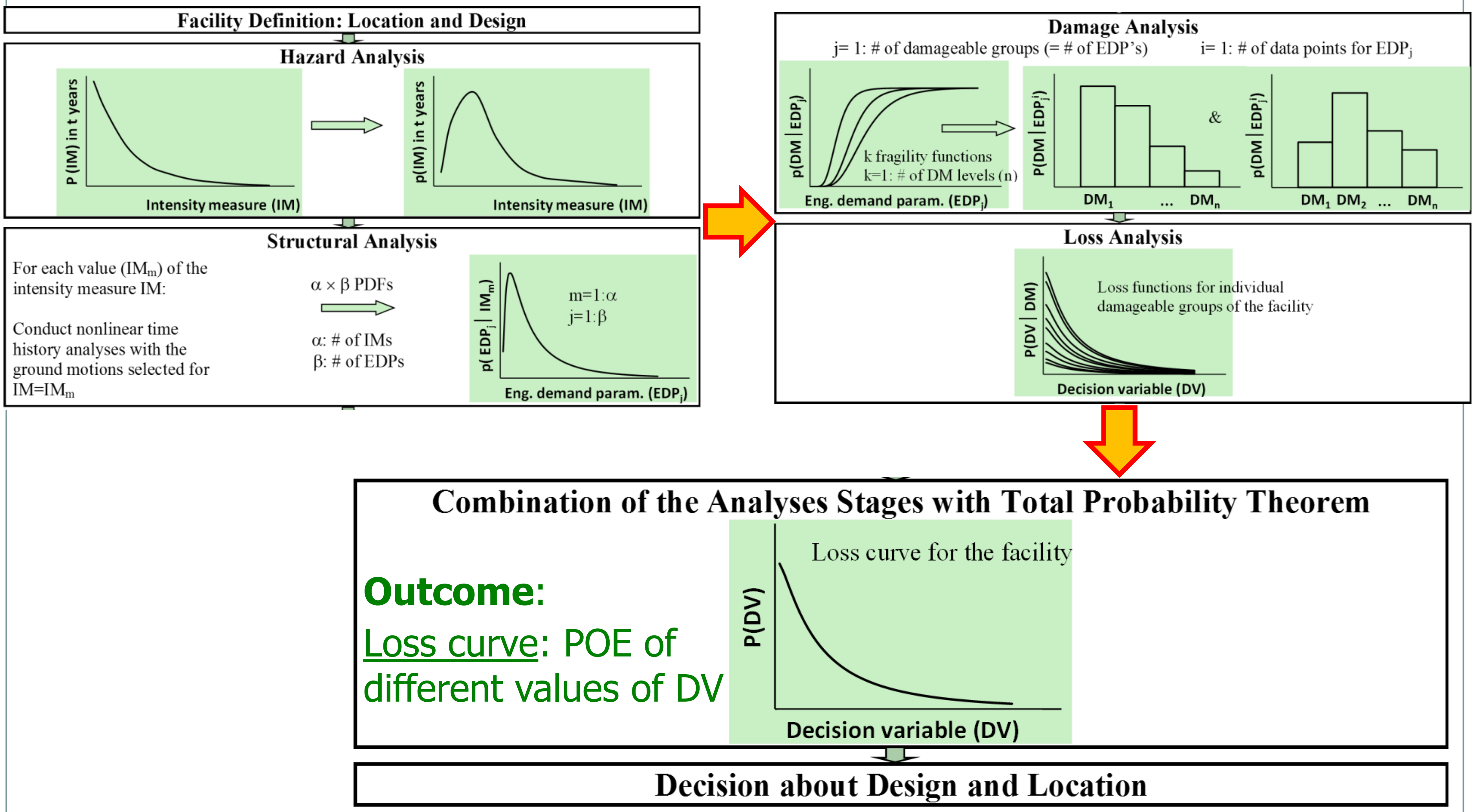
$$P(DV_j^n|EDP_j^i) = \sum_k P(DV_j^n|DM_k)\, p(DM_k|EDP_j^i)$$

Loss Analysis

Damage Analysis

- **m: index for IM**
- **j: index for damageable groups (DG)**
- **i: index for EDP**
- **k: index for DM**

Combination of Analyses



Combination of Analyses

Remark: *Loss*, *damage* & *structural* analyses results are summed in a straightforward manner. However, integration of the *hazard* analysis into the formulation does not take place in such a way because of the presence of damageable groups and collapse and non-collapse cases.

Straightforward equation in case of a single DG and no collapse:

$$P\left(DV^{n}\right)=\sum_{m}\sum_{i}\sum_{k}\underbrace{P\left(DV^{n}\middle|DM_{k}\right)}_{\text{Loss}}\underbrace{p\left(DM_{k}\middle|EDP^{i}\right)}_{\text{Damage}}\underbrace{p\left(EDP^{i}\middle|IM_{m}\right)}_{\text{Structural}}\underbrace{p\left(IM_{m}\right)}_{\text{Hazard}}$$

Direct resemblance to the PEER PBEE framework equation:

$$\lambda(DV)=\int\int\int G\langle DV\,|\,DM\rangle\, dG\langle DM\,|\,EDP\rangle\, dG\langle EDP\,|\,IM\rangle\, d\lambda(IM)$$

λ: Mean Annual Frequency (MAF), G: Conditional probability

Combination of Analyses

Remark: POE of the DV in case of collapse, P(DV|C), is not conditioned on the IM, whereas the POE of the DV in case of no collapse, $P(DV|NC,IM_m)$, is conditioned on the IM because:

- **No collapse** case consists of different damage states and the contribution of each of these damage states to this case changes for different IMs. This is not the situation for **collapse** case.
- For example, loss function for slight damage has the highest contribution from small values of IM, whereas the loss function for severe damage has the highest contribution from large values of IM.

Combination of Analyses

Variation in the formulation: Replace POE (**P**) with expected value (**E**)

Hazard Analysis

End product: $E(DV^n) = \sum_m E(DV^n | IM_m)\, p(IM_m)$

E of the nth value of the DV of the facility

Structural Analysis: Probability of no-collapse & of collapse

$$E(DV^n | IM_m) = E(DV^n | NC, IM_m)\, p(NC | IM_m) + E(DV^n | C)\, p(C | IM_m)$$

Loss Analysis: Loss function for collapse

$$E(DV^n | NC, IM_m) = \sum_j E(DV_j^n | NC, IM_m)$$

Structural Analysis

$$E(DV_j^n | NC, IM_m) = \sum_i E(DV_j^n | EDP_j^i)\, p(EDP_j^i | IM_m)$$

$$E(DV_j^n | EDP_j^i) = \sum_k E(DV_j^n | DM_k)\, p(DM_k | EDP_j^i)$$

Loss Analysis **Damage Analysis**

m: index for IM

j: index for damageable groups (DG)

i: index for EDP

k: index for DM

Combination of Analyses

Variation in the formulation: Replace POE (**P**) with expected value (**E**)

Loss Curve, POE of DV

$$P\left(DV^{n}\right)=\sum_{m}\sum_{j}\sum_{i}\sum_{k}P\left(DV_{j}^{n}\middle|DM_{k}\right)p\left(DM_{k}\middle|EDP_{j}^{i}\right)p\left(EDP_{j}^{i}\middle|IM_{m}\right)p\left(IM_{m}\right)$$

Expected Value of DV

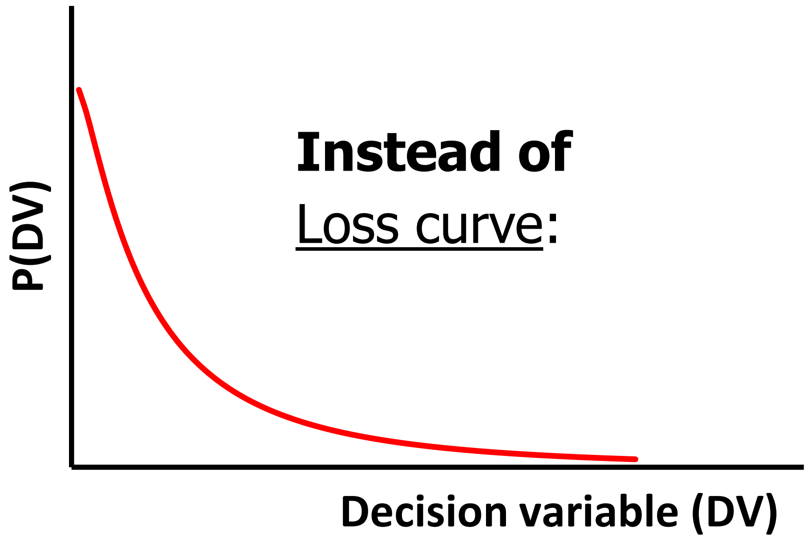
$$E\left(DV\right)=\sum_{m}\sum\sum\sum E\left(DV_{j}\middle|DM_{k}\right)p\left(DM_{k}\middle|EDP_{j}^{i}\right)p\left(EDP_{j}^{i}\middle|IM_{m}\right)p\left(IM_{m}\right)$$

Outcome:

Expected value of the decision variable

Instead of

Loss curve:



Combination of Analyses

Variation in the formulation: Consider a **single IM** value **(IM_1)**

Hazard Analysis

End product: **POE of the n[th] value of the DV of the facility**

$$P(DV^n) = P(DV^n | IM_1)\,[p(IM_1) \boxed{= 1.0}]$$

Structural Analysis: Probability of no-collapse & of collapse

$$P(DV^n | IM_1) = P(DV^n | NC, IM_1)\, p(NC | IM_1) + P(DV^n | C)\, p(C | IM_1)$$

Loss Analysis: Loss function for collapse

$$P(DV^n | NC, IM_1) = \sum_j P(DV_j^n | NC, IM_1)$$

Structural Analysis

$$P(DV_j^n | NC, IM_1) = \sum_i P(DV_j^n | EDP_j^i)\, p(EDP_j^i | IM_1)$$

$$P(DV_j^n | EDP_j^i) = \sum_k P(DV_j^n | DM_k)\, p(DM_k | EDP_j^i)$$

Loss Analysis **Damage Analysis**

m: index for IM = 1

j: index for damageable groups (DG)

i: index for EDP

k: index for DM

Combination of Analyses

Variation in the formulation: Consider a **single IM m**, p(IM_m)=1

Consideration of all possible hazard scenarios

$$P\left(DV^n\right)=\sum_m\sum_j\sum_i\sum_k P\left(DV_j^n\middle|DM_k\right)p\left(DM_k\middle|EDP_j^i\right)p\left(EDP_j^i\middle|IM_m\right)p\left(IM_m\right)$$

$$E\left(DV\right)=\sum_m\sum_j\sum_i\sum_k E\left(DV_j\middle|DM_k\right)p\left(DM_k\middle|EDP_j^i\right)p\left(EDP_j^i\middle|IM_m\right)p\left(IM_m\right)$$

Consideration of only specific hazard scenario

$$P\left(DV^n\right)=\sum_j\sum_i\sum_k P\left(DV_j^n\middle|DM_k\right)p\left(DM_k\middle|EDP_j^i\right)p\left(EDP_j^i\middle|IM_m\right)$$

$$p\left(IM_m\right)=1$$

$$E\left(DV\right)=\sum_j\sum_i\sum_k E\left(DV_j\middle|DM_k\right)p\left(DM_k\middle|EDP_j^i\right)p\left(EDP_j^i\middle|IM_m\right)$$

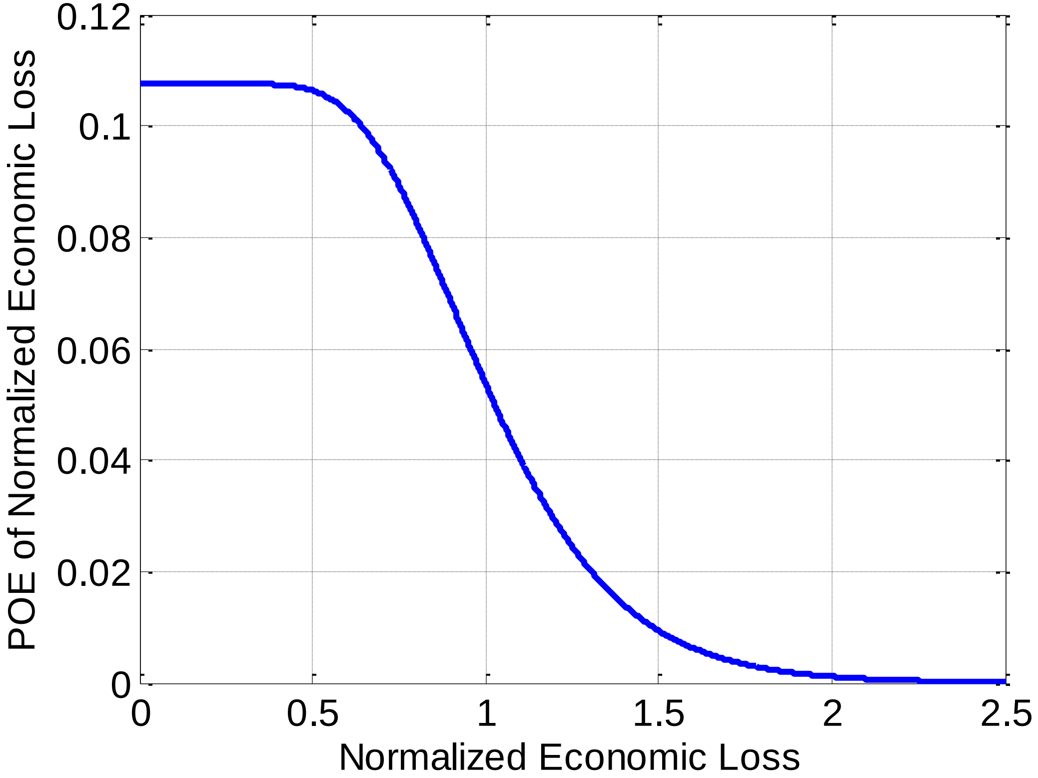
Combination of Analyses: Application

PEER PBEE: A powerful and efficient tool

- Not only for transformation of EDPs to meaningful DVs
- But also for investigating effect of various parameters on the seismic performance of disconnect switches
  - ✓ Support structure configuration
  - ✓ Insulator post type
  - ✓ Slack in the conductor cables
  - ✓ Location of the substation

Combination of Analyses: Application

Loss Curve

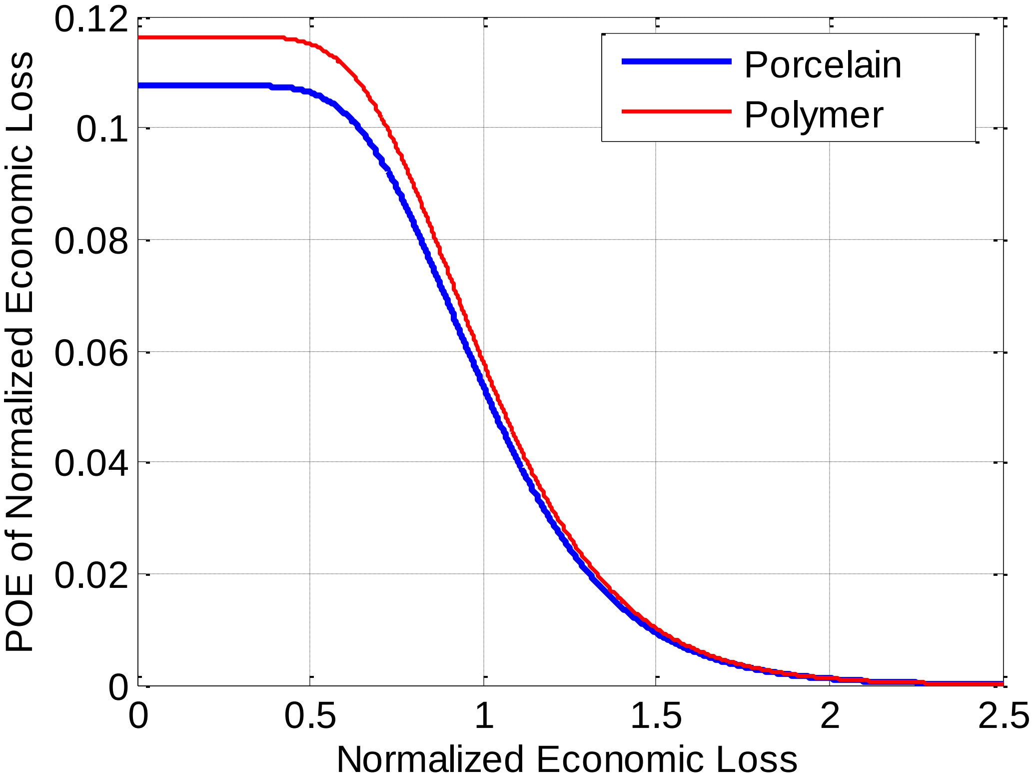


230 kV disconnect switch

- with porcelain insulators
- on a stiff braced support structure
- in Metcalf Substation

Combination of Analyses: Application

Loss Curve: Effect of Insulator Type

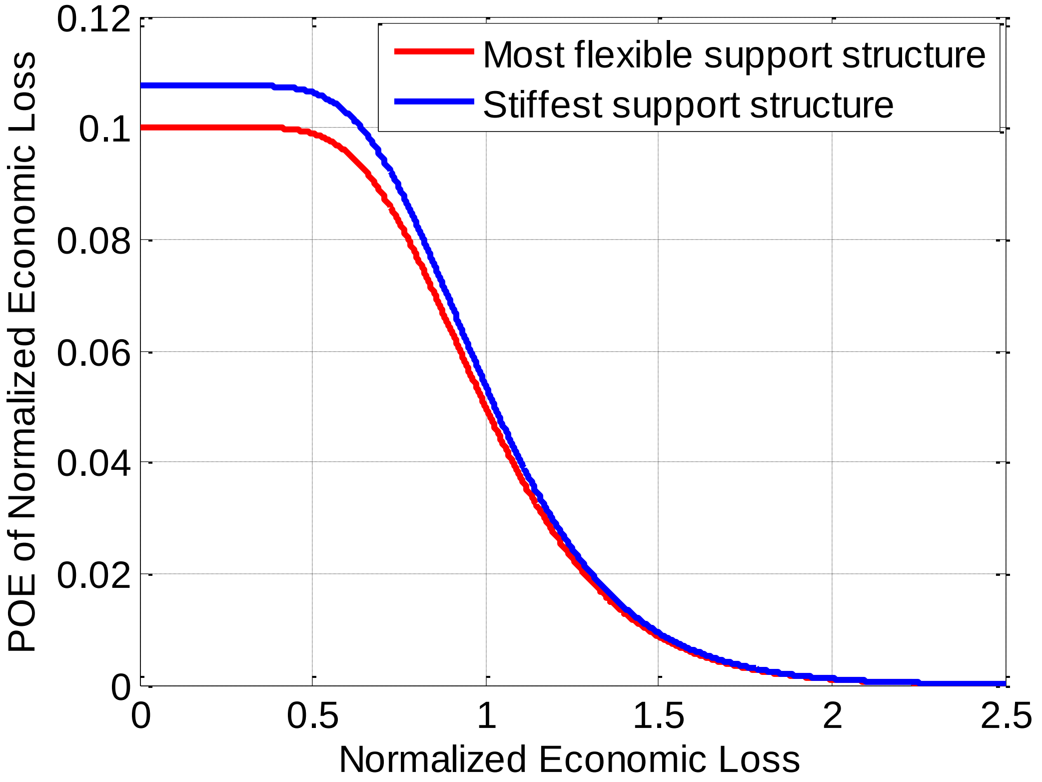


230 kV disconnect switch

- on a stiff braced support structure
- in Metcalf Substation

Combination of Analyses: Application

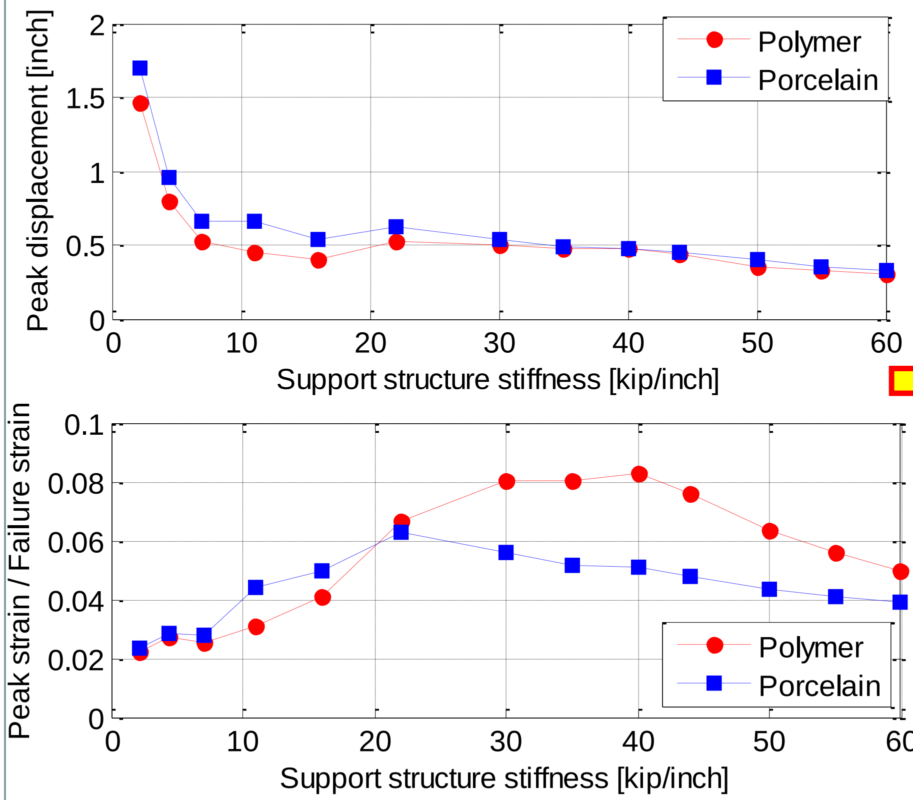
Loss Curve: Effect of Support Structure



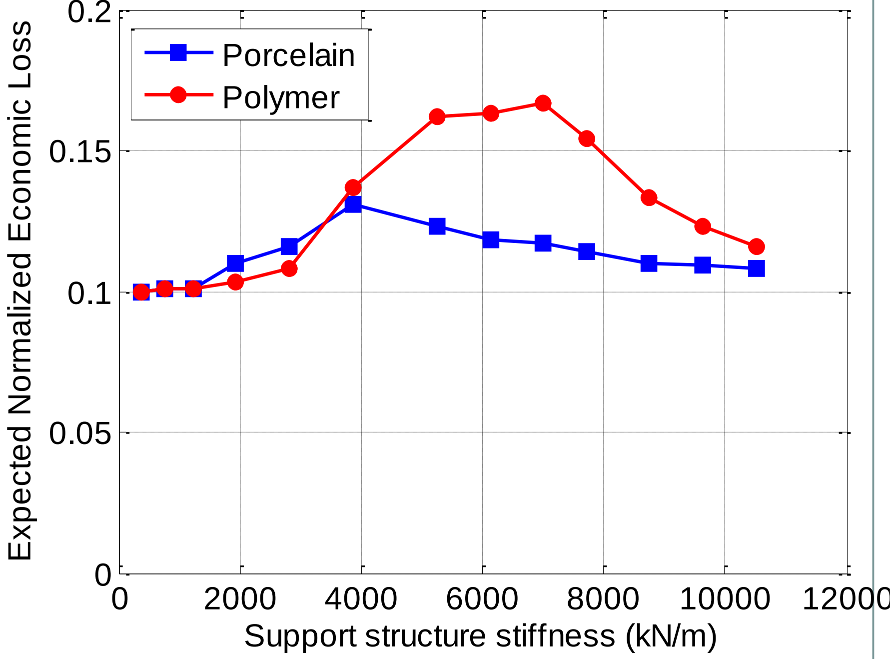
230 kV disconnect switch

- with porcelain insulators
- in Metcalf substation

Combination of Analyses: Application



Expected Loss: Effect of Insulator Type & Support Structure



The most flexible support structures, i.e. without braces, are the most suitable configuration for the investigated disconnect switches.

Combination of Analyses: Application

Effect of Slack Amount

1.65 x peak disp from RTHS: Ample slack

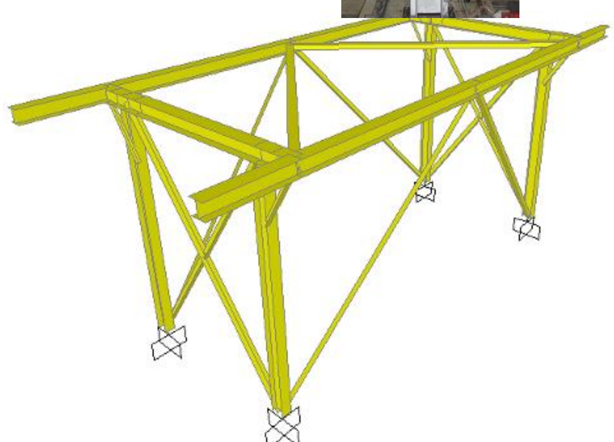
1.10 x peak disp from RTHS: Required slack

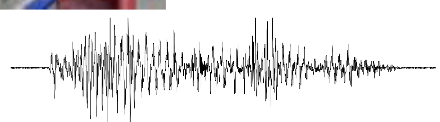
0.55 x peak disp from RTHS: Inadequate slack

Other equipment, e.g. transformer bushing

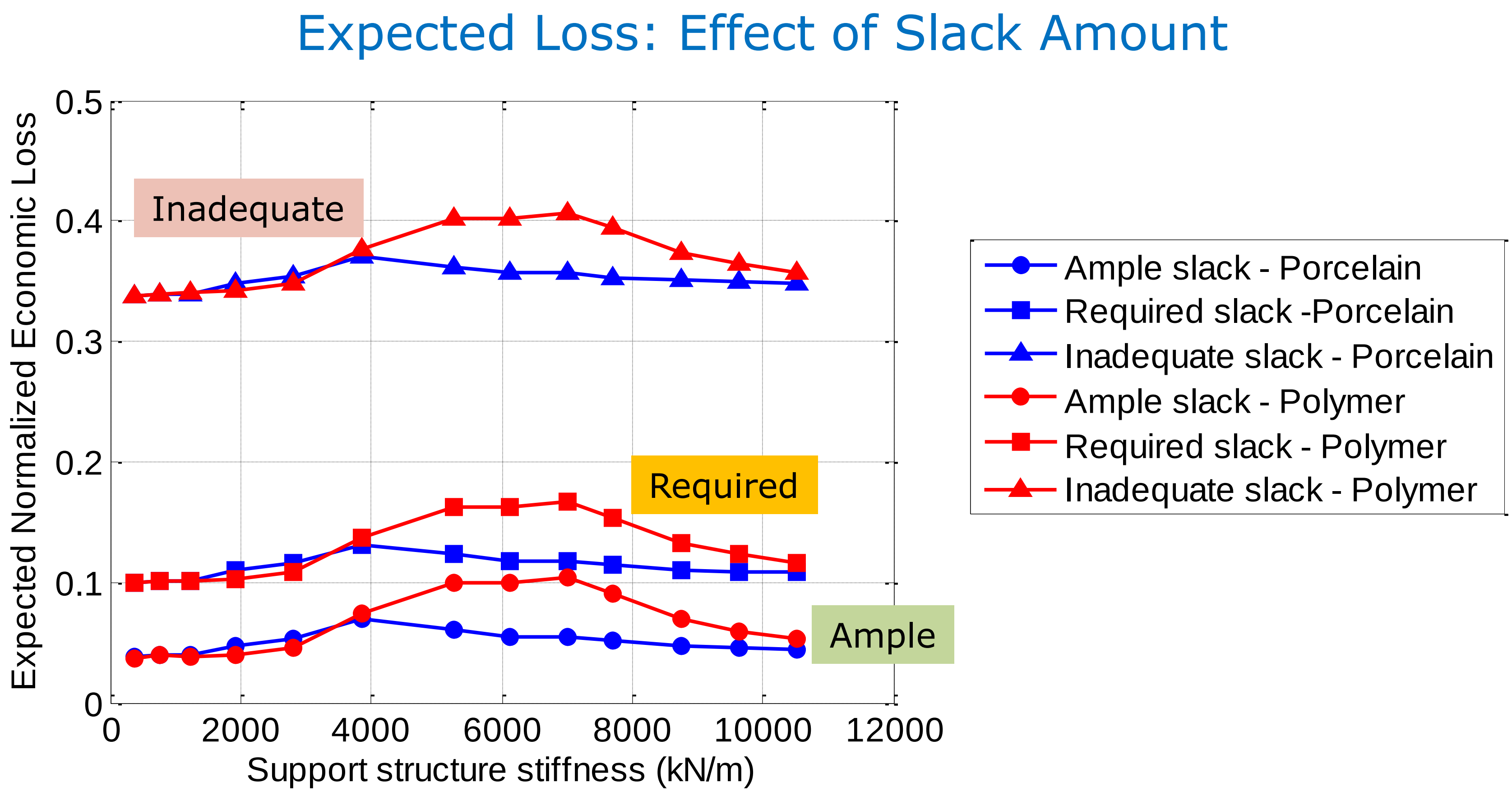
Disconnect switch

Slack amount





Combination of Analyses: Application



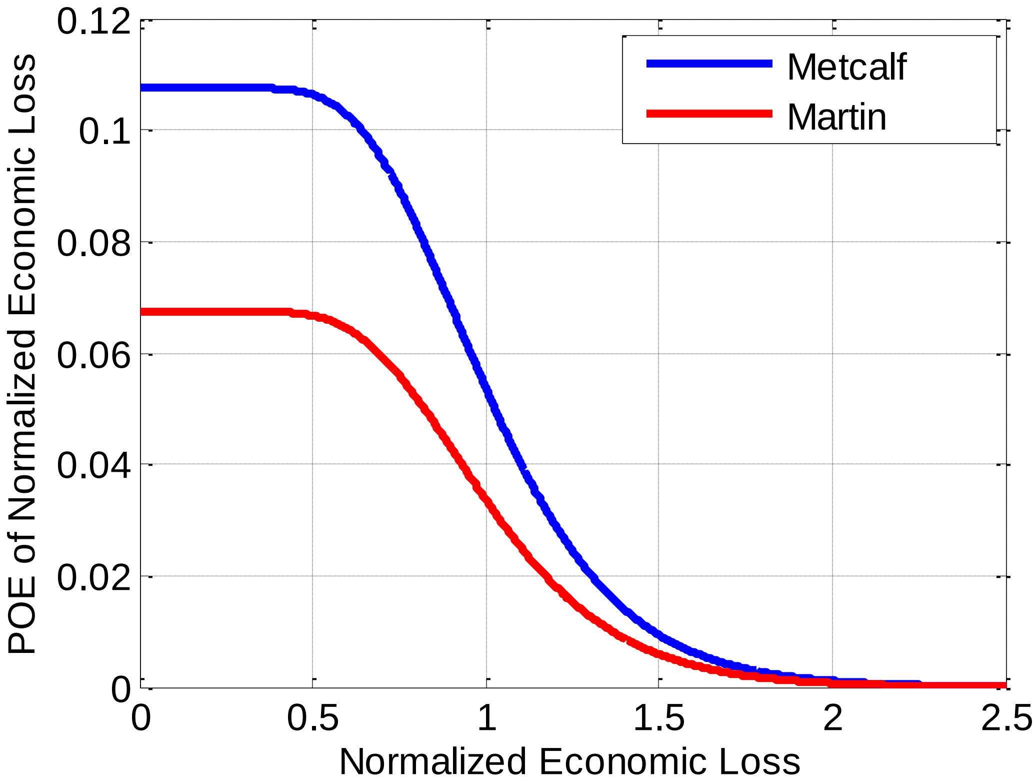
Slack amount has considerable effect on the expected losses

Combination of Analyses: Application

Martin Substation

Metcalf Substation

Loss Curve: Effect of Substation Location



230 kV disconnect switch

- with porcelain insulators
- on a stiff braced support structure

Application Options

How can an engineer use PEER PBEE method?

1. Evaluation of a traditional code-based design in a PBEE probabilistic approach. This application is appropriate in current state of traditional code-based design if the engineer wants to introduce performance-based enhancements to mandatory code-based design.
2. Evaluation of the performance of an existing structure or the outcome of different retrofit interventions.
3. Use of the methodology directly as a design tool, e.g. for decision-making amongst different design alternatives. This application is expected to gain widespread use when the probabilistic **PBED** methods start to be employed as a standard design method.

Questions?

mosalam@berkeley.edu

http://www.ce.berkeley.edu/people/faculty/mosalam